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When do Formal Rules and Informal Norms Converge?

by

DESIREE A. DESIERTO AND JOHN V. C. NYE*

We propose evolutionary dynamics to show how rules converge into norms. Individuals play a game of upholding or rejecting a rule, and the more they uphold the rule, the more it becomes established as a norm. We find that when individuals are rational, the initial state determines whether the rule converges into a norm; when individuals are boundedly rational, convergence occurs only if upholding rules is a risk-dominant strategy. This suggests why big-bang reforms that affect only the initial state can fail, while gradualist approaches that can sustain the risk dominance of upholding rules may be more effective. (JEL: P48, C73, B52)

1 Introduction

This paper proposes some evolutionary-game dynamics to describe how formal rules converge into informal norms, thereby providing a specific theory of institutional change that is rooted in the New Institutional Economics (NIE) literature. Scholars (e.g., Greif, Aoki, Young) have proposed a number of game-theoretic formulations to study institutional change, without coming to a clear consensus. Obviously, there are many types of institutions – some that are exogenous to individuals and some that individuals can alter by playing a game with other individuals – but even if one were to analyze only the latter type, the process is still unclear. One important reason is that while the literature acknowledges the role of random shocks and uncertainty in institutional change, it has yet to identify not only where the uncertainty comes from, but also how individuals react to this uncertainty. Until this issue is resolved, one cannot readily claim that institutional change is path-dependent. For instance, with persistent uncertainty, it could very well be that some institutions always tend to survive regardless of initial conditions.

In this paper, the kind of institutional change that we focus on is the process by which formal rules become established as informal norms as more and more individuals uphold them. Conversely, we look at how rules can die out *de facto*, even though they exist formally, as less and less individuals uphold these rules. We model

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this type of institutional change as a process of repeated strategic interaction, but interaction between individuals whose actions may not always be predictable, because individuals also react to (unpredictable) shocks. Thus, we use an evolutionary game that enables us to relax assumptions of hyperrationality implicit in other dynamic (i.e., repeated) games. By allowing the game to be played by individuals who can make mistakes, or *mutate* from expected behavior, we can gauge the strength of equilibria amidst noisy conditions, and are able to identify cases in which the convergence of formal rules into informal norms is approximately path-dependent and cases in which the initial conditions do not matter.

Thus, we first set out by treating formal rules as inherently exogenous to an individual, but allow the individual to endogenously determine the de facto existence of the rules by playing an evolutionary game of choosing whether or not to uphold such rules. The choice might only partly depend on the expected payoffs from upholding the rules, because in an uncertain environment, the individual might ignore expected payoffs (mutate) with some probability. In the aggregate, however, especially if a very large number of individuals play the game, such mutant behavior might be deemed trivial, in which case the population of individuals is approximately rational. Then again, it might be persistent, in which case, the population is (still) *boundedly rational*.

We find that whenever populations are (approximately) rational, the eventual outcome of the game depends only on the initial path. Specifically, if a sufficiently large number of individuals initially adopt the rules, then the expected payoff from upholding the rules becomes large enough that the rest of the population follow suit, and the rules become established as informal norms. However, when the population is boundedly rational, the initial path does not matter. Even if a large number of individuals initially disregard the rule, so that the expected payoff from upholding rules is small, the rule will still be established as a norm as long as upholding rules is a risk-dominant strategy for any type of player, rational or mutant. In this case, the reward for upholding rules is sufficiently high (and/or the punishment for not following rules is sufficiently high) that the strategy withstands even persistent mutant behavior.

Thus, the results suggest how big-bang approaches to instituting reforms can fail. To incentivize individuals to sustainably adopt rules, it might not be enough to effect large initial shocks that only alter the initial play. When mutations and mistakes are nontrivial, the strategy of upholding rules has to consistently pay off – at each point in time and for every type of individual.

The next section gives an intuitive exposition of the model, and section 3 formalizes. Section 4 interprets the results within the context of reform, and section 5 relates the model to other game-theoretic models in the NIE literature. Section 6 concludes.

2 Illustration

A driver reaches an intersection from which s/he intends to turn right, but the traffic light is red. Suppose the given rule is: “no right turn on red signal.” From the point

of view of the driver, this rule is predetermined, or exogenous, but s/he can choose to follow it or not. That is, by upholding the rule, s/he can (endogenously) determine whether that formal rule becomes, or converges into, an informal norm. To put it differently, s/he can influence the *de facto* existence of that rule – if s/he follows it, or *uses* the rule, then for that moment of usage, the rule exists *de facto*; if not, then it is nonexistent even though it formally exists. While the emergence of the formal rule may be exogenous to the driver, s/he can influence the extent to which it survives as an informal norm by choosing to uphold or reject the rule when s/he is faced with the opportunity.

Now the consequences of upholding or rejecting the rule depend on what type of individual or agent s/he interacts with, that is, whether that individual her/himself chooses to uphold or reject the same rule. Suppose the driver interacts with a traffic enforcer (police officer). The officer at that moment also chooses to uphold or reject the rule – upholding means that the officer will apprehend transgressors, while rejecting means s/he will not. Both driver and officer are better off by either both of them upholding or both of them rejecting the rule, than when one upholds it while the other rejects. When both uphold, the driver does not need to be apprehended, and the officer does not need to apprehend – that is, they avoid the cost of deviating from the rule. When both reject, the cost of deviation is avoided, since no apprehension occurs and, in addition, the driver arrives at her/his destination sooner by not stopping at a red light. However, when one upholds while the other rejects, there is a cost incurred by deviating from the rule or from miscoordinating actions (e.g., when the driver rejects while the officer upholds, the officer gives the driver a ticket; when the officer rejects while the driver upholds, the driver forgoes the time saved from not stopping at the red light).

Alternatively, the driver might interact with another driver, but the intuition is the same. If both drivers uphold the rule, then neither suffers a car accident (but nor do they save time from not stopping). If both reject the rule, there is also no accident, since both turn right (and both save time from not stopping).¹ But if one driver upholds the rule while the other rejects, an accident occurs (and no time is saved, or even more time is wasted).

Thus, to determine the *de facto* existence of a given (formal) rule at a given instant, a coordination game is played. As the game is (repeatedly) played during repeated interactions of individuals, the rule can become either more and more established as a norm, or weaker and less well established. Which equilibrium is reached for a given population depends on how members of that population generally behave in this repeated-interaction setting. For instance, if individuals were fully rational or behaved deterministically, then they could compute the expected of either strategy based on the history of play and choose the strategy with the higher expected payoff. It could then happen that if the rule is sufficiently established at the onset (i.e., a large

¹ Even if the other driver is coming from the left, if both reject the rule, there is still no accident. Rejection of the rule “no right turn on red signal” by the driver coming from the left means that this driver stops and gives way to the driver that is turning right.

enough number of individuals have already upheld it, so that the expected payoff of upholding the rule is now greater than that of rejecting it), then the next individuals will keep upholding it, which would increase the expected payoff of this strategy even more. When all individuals base their decisions only on expected payoffs, then the rule can become fully established in the population, depending on the initial play.

But what if some individuals are not rational, that is, they ignore expected payoffs? Think of a drunk driver who forgets that in the past other drivers that turned right from the same intersection on a red signal were punished – they either got caught or had a car accident – or an out-of-towner who does not know the traffic rules in that place. Or it could be any individual who is rational most of the time, but sometimes ignores expected payoffs when, say, in a hurry or emotionally wrought, or is sometimes absent-minded. In the presence of such mutant behavior, when rationality is bounded, there is the possibility that a rule weakens even when expected payoffs to upholding it are initially high, or strengthens even when expected payoffs to upholding it are initially low. Thus, the initial play may not matter, because there can always be a sufficiently large number of mutants whose random behavior can alter expected payoffs, thereby swaying the rest of the population. The final outcome then depends on which strategy, to uphold or to reject the rule, has stronger payoffs in each interaction or in every play of the game, so that, irrespective of the history of play and the current value of expected payoffs, that strategy continues to survive or grow. That strategy is then stable even in the presence of irrational or mutant behavior.

The same idea holds even if we consider not one, but a set of traffic rules that govern motorists' interactions. Suppose, in getting from point A to point B, motorists cannot (or do not have the time to) decide on each individual traffic rule, but take the rules in their entirety. That is, the motorist decides to uphold or to reject all relevant traffic rules in getting from point A to point B. The difference here is that since the action – getting from A to B – is now larger (more complex), each motorist may be more prone to mistakes in calculating the expected payoffs from upholding or rejecting the whole set of traffic rules. Moreover, even the smallest changes in the conditions underlying the complex interaction of going from point A to B can affect the ability of motorists to gauge expected payoffs and act rationally.² Thus, cases of complex actions, where rationality is arguably more bounded, provide even more convincing instances in which the initial play cannot sufficiently determine whether the (set of) rules become established as norms. What may be required are sufficiently large payoffs for upholding rules (or sufficiently small payoffs for not following rules) in every play of the game so that even mutants who reject rules when upholding them is rational according to current expected payoffs are still rewarded, while mutants who uphold rules when rejecting them is rational according to current

² For example, if road work or potholes were to appear along the way, these might confuse motorists and induce them to disregard rational calculations and act on their instincts.

expected payoffs are still penalized. In other words, the incentives do not only hold on expectation – that is, they are not only applicable to rational players, but are robust to mutant behavior as well.

3 The Model

In our model, (formal) rules become established as, or converge into, (informal) norms as more and more individuals adopt those rules whenever the opportunity arises. Faced with a specific interaction (e.g., at the intersection in the traffic-rule example) that is governed by a rule or set of rules, let agents of a population N be randomly drawn to play a pairwise (coordination) game $G_I = \{N, S, A\}$, where $S = \{1, 2\}$ is the strategy set common to all players, where $A = \{a_{ij}\}$, $i, j = \{1, 2\}$, is the corresponding (doubly symmetric) payoff matrix in which element a_{ij} is the payoff of adopting strategy i against an individual adopting strategy j :³

$$A = \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix},$$

and where strategy 1 is risk-dominant if $a_1 > a_2$.⁴ Let strategy 1 be to uphold or adopt the rule (or set of rules) while strategy 2 is to reject or ignore it. This coordination game G_I is what YOUNG [1993], [1998] also uses as the “stage” game for his (evolutionary-game) dynamic describing the establishment of norms, or what he terms “conventions.”

Let the outcome of each play of the game G_I be described by the proportions of players adopting each strategy as of time t . The state of the population at t can thus be described by $V = (v_1, v_2)$, where $v_1 = n_1/N$, and n_1 is the number of players that have adopted strategy 1 as of t (and $v_2 = 1 - v_1$ is the proportion that have adopted 2 as of t). Let payoffs be denoted generally as $u(e^1, e^1)$ for pure strategy 1 (and analogously for 2). The expected payoff from adopting strategy 1 when the population is in state V is then given by $u(e^1, V)$, and so the population average payoff is $u(V, V) = \sum v_i u(e^i, V)$, $i = \{1, 2\}$.

As $v_1 \rightarrow 1$, the rule (or set of rules) becomes more and more established as a norm (or set of norms), or, in other words, the (formal) rule converges to a (informal) norm. Let $\dot{v}_1$ capture the growth of strategy 1 such that whenever $\dot{v}_1 > 0$, v_1 increases towards one. (Note that $\dot{v}_1 + \dot{v}_2 = 0$, since $v_2 = 1 - v_1$.) In the following subsections, we propose some continuous-time *mean* dynamics $\dot{v}_1$ to approximate how v_1 grows in the population. That is, the dynamics closely capture, as $N \rightarrow \infty$, some underlying stochastic processes that specify how, at the micro level, each agent

³ A is constructed by subtracting a_{21} from column 1 and a_{12} from column 2. Note that $(a_{11} - a_{21})(a_{22} - a_{12}) > 0$.

⁴ For more on risk dominance, see HARSANYI AND SELTEN [1988]. Intuitively, if strategy 1, or upholding the rule, is risk-dominant, that means that rejecting, or deviating from, the rule gives higher payoffs (than upholding) only when others reject, or deviate from, the rule as well, but gives much lower payoffs when others choose to uphold. In this sense, upholding the rule is a safer bet and, on average, is less costly.

selects strategies. (See SANDHOLM [2010] for a discussion; also BOYLAN [1992], [1995], and SAMUELSON [1997].) We thus assume that the population N is very large.

3.1 Simple Rules

Consider first a simple interaction that is governed by a single rule. Suppose each agent selects between strategy 1 and strategy 2 in a manner that gives rise to the following *replicator dynamic*:⁵

$$(1) \quad \dot{v}_1 = [u(e^1, V) - u(V, V)]v_1,$$

by which a strategy grows when it does better than the population average. The dynamic (1) describes *unperturbed* behavior in the aggregate in that it is as if all players choose strategies only according to expected payoffs. That is, if $u(e^1, V) > u(V, V)$, the player adopts strategy 1 and v_1 grows. The standard result for 2×2 coordination games is that both strategies are asymptotically stable (AS), and the initial proportions determine which one of them is reached as $t \rightarrow \infty$. That is, using the particular payoffs A of the game G_I :

LEMMA 1 *For a large enough initial proportion $v_1^0 > a_2/(a_1 + a_2)$, one has $v_1 \rightarrow 1$, but for $v_1^0 < a_2/(a_1 + a_2)$, $v_1 \rightarrow 0$.*

Lemma 1 implies that if the process of convergence of the formal rule and informal norm can be approximated by the dynamic (1), then the process is (fully) path-dependent in the sense that the system converges to either $v_1 = 0$ or $v_1 = 1$ as $t \rightarrow \infty$, depending on the initial condition v_1^0 .

Now suppose expected payoffs are ignored to some extent. Specifically, suppose a player can mutate, or randomly switch between strategies, and denote by λ_{ij} the probability of switching from strategy j to i . Analogously to FUDENBERG AND HARRIS [1992] and CABRALES [2000], let the mutation flow to v_i be captured by $\sum_{j=1}^2 [\lambda_{ij}v_j - \lambda_{ji}v_i]$.⁶ Letting $\lambda_{ij} = \lambda_{ji} = \lambda$ and using the fact that $v_2 = 1 - v_1$ so that $\sum_{j=1}^2 [\lambda_{ij}v_j - \lambda_{ji}v_i] = \lambda(1 - 2v_1)$, suppose such a process of selection of strategies with fixed probability of mutation (switching) can be approximated, as $N \rightarrow \infty$, by the following *perturbed* mean dynamic:

$$(2) \quad \dot{v}_1 = [u(e^1, V) - u(V, V)]v_1 + \lambda(1 - 2v_1),$$

where the net inflow into strategy 1 is composed of the flow according to expected payoffs and the mutation flow. Samuelson provides other forms of perturbed selection dynamics (based on the aspiration-and-imitation model), but the general idea is

⁵ See, for instance, TAYLOR AND JONKER [1978], WEIBULL [1995], VEGA-REDONDO [1996], Samuelson, and Sandholm on how the RD is explicitly derived. We focus on the RD for strategy 1; the RD for 2 is easily obtained from the fact that growth rates sum to zero.

⁶ In Fudenberg and Harris and in Cabrales, the mutation flow is expressed in terms of the number of players, i.e., $\sum_{j=1}^2 [\lambda_{ij}n_j - \lambda_{ji}n_i]$, and not in terms of proportions.

that some proportion of the population evolve according to the unperturbed dynamic while some are subject to perturbations.

With such perturbations, the system never converges to any point. However, it spends most of its time near the risk-dominant equilibrium. That is, we find that the risk-dominant strategy is selected as the equilibrium strategy in the following sense:

LEMMA 2 *For any value $\lambda \in (0, 1)$ and initial proportion v_1^0 , v_1 never converges to any point but is almost always within the basin of attraction of the risk-dominant strategy.*

PROOF We give a rough proof. Using the payoff matrix A , suppose strategy 1 is risk-dominant, and let $a_2/(a_1 + a_2)$ denote the mixed Nash equilibrium (NE) such that the basin of attraction of strategy 1 is $(a_2/(a_1 + a_2), 1)$. Note that the mixed NE is less than $1/2$, since $a_1 > a_2$. That is, the basin for 1 is larger than $(0, 1/2)$. From (2), we can see that for any value $\lambda \in (0, 1)$, v_1 grows (i.e., $\dot{v}_1 > 0$) as long as

$$\lambda > \frac{-[u(e^1, V) - u(V, V)]v_1}{1 - 2v_1} \equiv \xi$$

and decreases when the inequality is reversed. Since 1 is risk-dominant, the numerator of ξ is always negative when $a_2/(a_1 + a_2) < v_1^0 \leq 1$. The denominator is positive so long as $v_1 \leq 1/2$ and hence $\xi < 0$, and v_1 keeps increasing, since any value of λ will always be greater than ξ . When $v_1 > 1/2$, we have $\xi > 0$, making the inequality

$$\lambda > \frac{-[u(e^1, V) - u(V, V)]v_1}{1 - 2v_1}$$

binding, but the point at which v_1 starts decreasing is always above $1/2$ and, hence, above $a_2/(a_1 + a_2)$. (At $v_1^0 = 1$, we have $\dot{v}_1 = -\lambda$, so v_1 decreases, and the system is deflected from the boundary $v_1 = 1$.) On the other hand, when $v_1^0 < a_2/(a_1 + a_2)$, the numerator of ξ is positive while the denominator is positive, so the inequality is binding and v_1 keeps increasing. (At $v_1^0 = 0$, we have $\dot{v}_1 = \lambda$, so v_1 increases, and the system is deflected from the boundary $v_1 = 0$.) Thus, for any v_1^0 , v_1 is almost always within the basin $(a_2/(a_1 + a_2), 1)$, since it only decreases somewhere in $(1/2, 1] \subset (a_2/(a_1 + a_2), 1)$, and increases everywhere else. *Q.E.D.*

An analogous result is found even in explicitly stochastic models, as in Cabrales, in Fudenberg and Harris, in KANDORI, MAILATH, AND ROB [1993] (hereafter KMR), and in Young. That is, with mutation, the system is always deflected from the boundaries – or from any interior point, for that matter – and hence never converges to any point. Moreover, those authors show that as the probability of mutation goes to zero, the (ergodic) distribution of strategies collapses to a point mass on the risk-dominant strategy. In this sense, the risk-dominant strategy is stochastically stable (SS).

BERGIN AND LIPMAN [1996] show that this result is generated because mutation rates between strategies are the same (“state-independent”). With the same probability of mutation out of either strategy shrinking at the same rate, it becomes more difficult to mutate out of the basin of attraction of the risk-dominant strategy,

since this basin is larger (deeper). Thus, when mutation shrinks to zero, one finds that the distribution puts probability 1 on the state in which everyone adopts the risk-dominant equilibrium.

The evolution of a formal rule into an informal norm can thus be governed by either dynamic (1) or (2), depending on the extent to which agents decide according to expected payoffs.

Lemmas 1 and 2 thus prove the following result:

PROPOSITION 1 *If agents are rational, a simple rule eventually becomes established as a norm if an initially large enough number of agents simultaneously uphold it. If agents are boundedly rational, the simple rule tends to be established as a norm if it is a risk-dominant strategy to uphold it.*

Note that both dynamics (1) and (2) are still deterministic, albeit some mutation behavior is allowed. We show, however, that the results of Proposition 1 are maintained even when one explicitly adopts a stochastic dynamic. The next section formulates such a dynamic, and shows how the latter can also approximate the deterministic dynamics (1) and (2) under certain conditions.

3.2 Complex Rules

Some interactions are *complex* in that they are governed not just by one rule but by a set of rules, e.g., a single treaty composed of many rules, a bill composed of many provisions, or a set of interconnected traffic rules to get from location A to B. Such complex rules become norms only when they are adopted in their entirety. Agents then still play the game G_I , but strategy 1 now refers to upholding the entire set of rules while strategy 2 is to reject the set. One might also expect that unlike simple interactions, complex interactions, and hence complex rules, are more likely to be affected by the slightest changes in the environment or underlying conditions, precisely because there are many components (elements) in a complex rule.⁷ Changes in a single environment can affect these multiple rules, and the slightest change affecting just one rule can render the entire complex rule unstable. Thus, in capturing the evolution of complex rules, one needs to explicitly take into account changes in the environment.

This is not to say that simple rules are not affected by changes in the environment, but one can arguably abstract from the effects of the environment and approximate the dynamic by either (1) and (2) if the rule is simple ("small") enough. In fact, in

⁷ The NIE literature suggests that institutions are formed in order to adapt to the environment or underlying conditions (in which the interaction occurs). According to NORTH [1994, p. 363], institutions serve as "external mechanisms individuals create to structure and order the environment." This notion is also implicit in the empirical debate on whether geography affects economic performance directly or indirectly via institutions. (See RODRIK, SUBRAMANIAN, AND TREBBI [2004], EASTERLY AND LEVINE [2003], SACHS [2003], ACEMOGLU, JOHNSON, AND ROBINSON [2002], [2001], and MCARTHUR AND SACHS [2001].)

this subsection we show explicitly how (1) and (2) can still be derived from a more general (stochastic) dynamic.

Specifically, suppose the environment or underlying conditions are subject to (continuous) random shocks captured by a Brownian motion $\dot{q}$ that describes the flow of a Wiener process W_t , which is distributed as normal $(0, t)$ and whose increments are independent and normally distributed with mean 0. (That is, for $0 \leq s < t$, the random variable $W_t - W_s$ is distributed as normal $(0, t - s)$.) Using Brownian motion to approximate the flow of shocks allows us to consider very small (i.e., continuous) changes to the environment, and to specify such a flow as one with mean zero, which would make the environment or underlying conditions fixed on expectation.⁸ Lastly, suppose positive shocks $\dot{q} > 0$ have the effect of “expanding” the environment (making it more complex), while negative shocks $\dot{q} < 0$ have the effect of “contracting” the environment (making it less complex). The shocks, in turn, can partly determine whether individuals choose to adopt the rules.

Without providing a similar interpretation for Brownian motion, FOSTER AND YOUNG [1990], Fudenberg and Harris, and Cabrales nevertheless use it to capture random flows into and out of strategies. Foster and Young analyze the following dynamic:⁹

$$\dot{v}_1 = v_1 \{[u(e^1, V) - u(V, V)] + \sigma \Gamma(V) \dot{q}\},$$

where $\Gamma(V)$ is continuous in the state space V , and $V^T \Gamma(V) = [0, 0]^T$, and σ is some arbitrarily small quantity. Considering a small interior subset of the state space and assuming that the disturbance is uniform over the state space and over time, Foster and Young approximate the behavior of the above dynamic by illustrating the evolution of v_1 according to

$$\dot{v}_1 = [u(e^1, V) - u(V, V)]v_1 + \sigma \dot{q},$$

where the second term captures a random drift into strategy 1.

In our version, there are also players that randomly drift into strategy 1, but there are also those that switch (mutate) between strategies 1 and 2. More importantly, the probability of mutation is not fixed, but evolves according to $\dot{q}$:

DEFINITION 1 (STOCHASTIC MUTATION RATE) *Let the rate of mutation for both player types be given by $\lambda \equiv e(\dot{q})$, and, defining $Q \subset \Re$ as the set of all values for $\dot{q}$, let $e : Q \rightarrow [0, 1]$, $e' > 0$, $e'' = 0$, and $e(0) \geq 0$.*

That is, agents tend to experiment more, or are less confident of their strategy and hence mutate more, as they experience more random shocks as captured by $\dot{q}$. Positive (negative) shocks to the environment, which make the latter more (less) complex, induce greater (less) mutation. With shocks $\dot{q}$ and stochastic mutation rate

⁸ Whether indeed this is the case is an empirical issue. Arguably, however, the environment cannot be so unstable as to undergo persistently positive (or negative) shocks. That is, on average, the environment is fixed.

⁹ Unlike Foster and Young, Fudenberg and Harris and Cabrales add the Brownian-disturbance term to the growth of strategies in terms of number of players, not to the growth of proportions.

$\lambda \equiv e(\dot{q})$, assume that the underlying selection process can thus be approximated, as $N \rightarrow \infty$, by the following *mean dynamic*:¹⁰

$$\dot{v}_1 = [u(e^1, V) - u(V, V)]v_1 + \sigma\dot{q} + e(\dot{q})(1 - 2v_1),$$

or, using payoffs for the game G_I ,

$$(3) \quad \dot{v}_1 = [a_1v_1 - a_1v_1^2 - a_2(1 - v_1)^2]v_1 + \sigma\dot{q} + e(\dot{q})(1 - 2v_1).$$

By the dynamic (3), the evolution of complex rules into (complex) norms is driven by three things. The first term describes the part that is governed by deterministic (and unperturbed) evolution, while the second and third are stochastic components driven by the random evolution $\dot{q}$ of the environment, which induces, as it were, first- and second-order noise in the dynamic. In the first place, as the environment becomes larger, i.e., $\dot{q} > 0$, there is a positive flow $\sigma\dot{q}$ into v_1 . This reflects some amount of positive bias or “optimism” for the complex rule, irrespective of what the actual expected payoffs are. The idea is that, all else being equal, a larger environment will have greater need for rules that can structure it. Secondly, a complex environment also induces uncertainty about expected payoffs, which is manifested as switching, or mutation, between strategies. The larger the shocks, or the larger the environment, the greater the probability of mutation into and out of strategy 1. The intuition is that agents become more unsure about their evaluation of the complex rule as the environment becomes larger.

Thus, positive shocks $\dot{q} > 0$ may tend to increase random drift into v_1 (and negative shocks $\dot{q} < 0$, out of v_1), but they can also increase (decrease) the probability of switching from strategy 1 to 2. The net effect is not clear – the behavior of the dynamic (3) as $t \rightarrow \infty$ has yet to be established. On one hand, one can appeal to a result obtained by ROBLES [1998], which shows that with time-varying mutation, if $\lambda_t \rightarrow \hat{\lambda} > 0$ (where $\hat{\lambda}$ is some fixed rate to which the time-varying rate will converge), and if a stationary distribution exists for the process defined by $\hat{\lambda}$, then a (modified) long-run equilibrium (MLRE) exists. This MLRE just coincides with the equilibrium of the process defined by fixed $\hat{\lambda}$. (This would then imply that the risk-dominant equilibrium would be MLRE.) In our model, however, the only way for λ to converge to some fixed value is to have $\dot{q}$ converge to a fixed value, and even then the drift term would still be nonzero (unless $\dot{q} \rightarrow 0$).¹¹

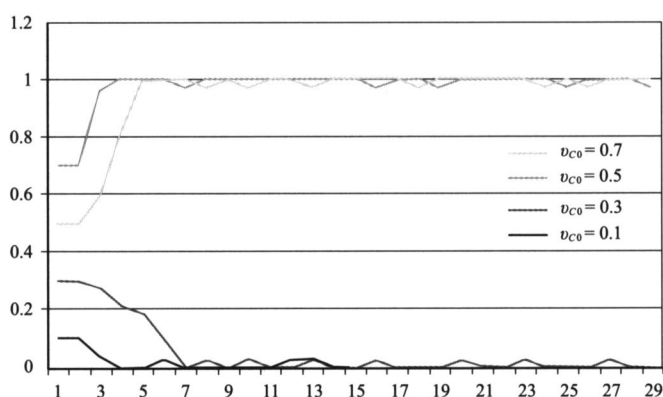
¹⁰ That is, (3) simply adds to the RD a Brownian motion as drift term and the mutation flow in (2), but with $\lambda \equiv e(\dot{q})$.

¹¹ Note, though, that Robles does not explicitly model drift (nor does he relate mutation to drift). By a different method, viz., computing a compound probability distribution, HOUBA AND TIEMAN [2001] also show that the equilibrium obtained by KMR (i.e., the risk-dominant strategy) coincides with the equilibrium obtained from recasting KMR using time-varying mutation rates. Like Robles, however, Houba and Tiesman do not explicitly consider (Brownian) drifts.

How might (3) behave while $\dot{q}$ is not fixed? The following example shows how, depending on the initial value v_1^0 , the system could keep fluctuating, but it does so either near $v_C = 1$ or near $v_C = 0$.

Example. We give a rough approximation of the behavior of the dynamic (3) by simulating in discrete time the path of the proportion $v_{Ct} = v_{1t} = v_{1t-1} + \Delta v_{1t-1}$ when strategy 1 is assumed to be risk-dominant, the function e takes the simple form $\lambda_t = \epsilon q_t = \epsilon(q_{t-1} + \Delta q_t)$, and Δq is drawn at each t from a uniform distribution $U(-1, 1)$.¹² The figure shows the evolution of $v_C = v_1$ assuming different initial values, and it is seen that the dynamic initially approaches either $v_C = 1$ or $v_C = 0$ and stays nearby, even as it is continuously deflected from the boundary.

Figure
Dependence on Initial Value



The above example suggests that neither boundary is an asymptotically stable state, but once the system enters its vicinity, it stays (albeit fluctuating) within it. It is deflected from the boundary because whenever $\dot{q} > 0$ the mutation outflow is larger than the mutation inflow as $v_1 \rightarrow 1$, and thus, outward mutation deflects the system from $v_1 = 1$. When $\dot{q} < 0$ the mutation outflow is smaller than the mutation inflow, but now it is the negative drift that deflects the system from $v_1 = 1$. However, the system fluctuates near the boundary, because once the system is in the basin of $v_1 = 1$, it becomes harder for negative drifts out of strategy 1 to move the system to the other basin $v_1 = 0$. The reason is that not only is the basin $v_1 = 0$ smaller, but a negative drift out of $v_1 = 1$ also increases the mutation inflow into strategy 1

¹² The expected value of Δq is thus 0. We also assume initial $q_0 = 3$, $a_1 = 2$, $a_2 = 1$, $\sigma = 0.03$, and $\epsilon = 0.0001$.

(while a positive drift into $v_1 = 0$ increases the mutation *outflow* from 2.)¹³ That is, all else being equal, it would take a larger negative drift out of $v_1 = 1$ to offset the corresponding drift into $v_1 = 0$, because the latter is also mitigated by a larger mutation *outflow* from strategy 2. Thus, while nonvanishing noise prevents v_1 from asymptotically approaching 1 as $t \rightarrow \infty$, v_1 still stays fluctuating around $v_1 = 1$ once it enters its basin, that is, if its initial value v_1^0 is large enough.

To generalize the behavior of the dynamic (3), we use an approach that may be intuitively analogous to that of Robles (and Houba and Tieman) by considering the *average* dynamic in which the noise behaves as expected, and the resulting dynamic becomes deterministic. Note that the expected value of $\dot{q}$ is zero. Letting $\dot{q}$ converge to its expected value thus lets the drift term disappear and lets mutation rates be fixed.

To motivate this, note that any single environment can have very many complex rules, where each complex rule $r = 1, 2, \dots$ evolves according to the dynamic (3), that is, according to the same process $\dot{q}$. Then, any complex rule in that same environment is expected to evolve as the expected value $\dot{v}_1 = E(\dot{v}_1 | v_1)$, where the expectation is taken over the distribution of $\dot{q}$. That is, letting $\dot{v}_1 = E(\dot{v}_1 | v_1)$, suppose that the underlying stochastic selection process for any complex rule can be approximated, as $N \rightarrow \infty$ and $r \rightarrow \infty$, by the following average (mean) (deterministic) dynamic:¹⁴

$$\dot{v}_1 = E\{[a_1 v_1 - a_1 v_1^2 - a_2(1 - v_1)^2]v_1 | v_1\} + E[(\sigma \dot{q}) | v_1] + E\{[e(\dot{q})(1 - 2v_1)] | v_1\}.$$

Assuming independence of $\dot{q}$ from past and current v_1 ,¹⁵ the above reduces to

$$\dot{v}_1 = [a_1 v_1 - a_1 v_1^2 - a_2(1 - v_1)^2]v_1 + \sigma E(\dot{q}) + E[e(\dot{q})(1 - 2v_1)].$$

Since $\dot{q}$ describes Brownian motion, $E(\dot{q}) = 0$, and the above simplifies to

$$(4) \quad \dot{v}_1 = [a_1 v_1 - a_1 v_1^2 - a_2(1 - v_1)^2]v_1 + e(\bar{q})(1 - 2v_1),$$

where $\bar{q} = E(\dot{q}) = 0$. When $e(\bar{q}) = 0$, then the above becomes the same as (1) and behaves exactly like the unperturbed RD where agents are rational, and Lemma 1 applies. If $e(\bar{q}) > 0$, the above has the form of the dynamic (2) where agents are boundedly rational, and Lemma 2 applies. Thus, on expectation, even complex rules can evolve like simple rules. That is, we can generalize Proposition 1 to the case of complex rules:

PROPOSITION 2 *If agents are rational, a complex rule eventually becomes established as a norm if an initially large enough number of agents simultaneously uphold it. If agents are boundedly rational, the complex rule tends to be established as a norm if it is a risk-dominant strategy to uphold it.*

¹³ Note that a (negative) drift out of $v_1 = 1$ (or strategy 1) is equivalent to a (positive) drift into $v_1 = 0$ (or strategy 2), and mutation inflow into strategy 1 corresponds to mutation outflow from strategy 2.

¹⁴ Recall that the dynamic (3) is already an approximation, or a mean dynamic, for very large N . “Average” refers to the fact that (4) approximates the evolution of any complex rule in the same environment, given very large r .

¹⁵ That is, changes in q affect the next and future values of v_1 through drift and mutation, but not its current or past values.

4 Interpretation

Section 3 provides some dynamics that can approximate the aggregate behavior of a population of agents who choose to either adopt or reject a given (simple or complex) formal rule. If agents are (approximately) rational, then the evolution of formal rules into informal norms or conventions may roughly follow (1). On the other hand, if agents' rationality is significantly bounded,¹⁶ then the dynamic (2) may describe the process more appropriately. Thus, for any (simple or complex) rule, Propositions 1 and 2 identify the general conditions under which rules become, or converge to, norms. Under the assumption of rationality of agents, convergence depends on the initial path. That is, with sufficient momentum – i.e., a large enough number of players ($v_1^0 > a_2/(a_1 + a_2)$) adopting the rule simultaneously – the rest of the population follow suit. However, under bounded rationality, convergence occurs if upholding rules is a risk-dominant strategy for any and every player.

This implies that in order to ensure convergence when agents are rational, one only needs to alter the (initial) population profile, while the case of boundedly rational players requires a more fundamental change – that of adjusting the payoffs of the game itself. Note that v_1^0 need not be so high in order to satisfy $v_1^0 > a_2/(a_1 + a_2)$ if upholding rules is a risk-dominant strategy, since, with $a_1 > a_2$, $a_2/(a_1 + a_2)$ is smaller. However, when upholding rules is not a risk-dominant strategy, i.e., $a_1 < a_2$, one needs a larger shock v_1^0 in order to satisfy $v_1^0 > a_2/(a_1 + a_2)$ (since $a_2/(a_1 + a_2)$ is larger).

The foregoing suggests that, without knowing the true extent of agents' rationality, a more effective way of transforming formal rules into norms would be to establish the risk dominance of upholding rules. If agents turned out to be boundedly rational, then our model predicts that convergence would be attained (or at least approximated), despite the mutations or mistakes that individual players might make along the way. If agents were rational, then convergence would also likely be attained in the sense that for any initial path v_1^0 , that is, without deliberately affecting the size of v_1^0 , it could easily happen that $v_1^0 > a_2/(a_1 + a_2)$ is met (since $a_2/(a_1 + a_2)$ is small). Thus, in terms of policy actions, targeting the size of payoffs from upholding rules, rather than imposing one-time shocks that force agents to immediately uphold the rule, may be a more effective strategy to ensure overall de facto compliance with the rule or, equivalently, the establishment of the rule as a norm. In the first place, even if agents were rational, the shocks might not be sufficiently large, that is, it might be difficult to calculate and effect $v_1^0 > a_2/(a_1 + a_2)$. Secondly, if agents turned out to be boundedly rational, then no amount of shock or influence on v_1^0 would establish the rule as a norm if upholding rules did not bring high enough payoffs (and/or deviating from the rules did not generate low enough payoffs), so that upholding rules was not a risk-dominant strategy.

¹⁶ That is, if individual mistakes (mutations) of players carry over to the aggregate, so that the appropriate mean dynamic for the population is given by (2).

This helps to explain why reforms can fail even if resources are poured into staging large coordinated efforts at the initial stages. Big-bang approaches to reform can fail in two ways: either the bang may not be big enough, so that even fully rational agents are not spurred to adopt the reform; or, even if the bang is very big, the payoffs to upholding rules may not be large enough to withstand persistent mutations or mistakes of some agents. Since, with the possibility of mutation, agents do not always end up following the rest of the population, the only way to ensure that the rule is generally upheld is to sufficiently penalize deviation from the rule (or sufficiently reward compliance with the rule) at the micro level, that is, for every player or whenever the game is played. In this sense, a more gradual approach to reform that sustains the risk dominance of upholding rules at the micro level is more likely to be successful.

The notion of risk dominance and the implication that sustained efforts (as opposed to one-time, big-bang actions) are more likely to achieve it also have broader applications in social science research. For instance, rational-choice theories of group solidarity point out that continuous reinforcement of norms is needed to sustain long-term patterns of behavior (HECHTER [1987]). In other forms of socialization, the sustained effort required might come in the form of effective monitoring (OLSON [1965]). Finally, the importance of continuous monitoring and enforcement costs in establishing norms also ties into the club literature on the constraints of large group size (BUCHANAN [1965]).

5 *Related Literature*

GREIF [2006, p. 8] notes that while some institutions can be thought of as formal rules or laws, they are only viable if they lead to compliance: “If prescriptive rules of behavior are to have an impact, individuals must be motivated to follow them.” He then demonstrates the usefulness of classical game theory in showing how institutions endogenously emerge as stable equilibria as individuals respond *de facto* to formal rules.

In addition, by particularly using results from dynamic (i.e., repeated) games, Greif captures the notion of path dependence in institutional change. This is important, since path-dependent stories typically give greater weight than static analyses to the possibility that initial events can sway long-run outcomes. However, Greif’s framework is still incomplete, because it does not explicitly consider how random shocks might affect the behavior of agents, or how agents might sometimes act irrationally. Without explicitly modeling the role of noise in the process of institutional change, the classical framework cannot gauge the true extent of path dependence, nor the long-run stability and viability of outcomes.

In this sense, AOKI [2009], [2007] is more encompassing than Greif in that he does not eschew the role of mutations or bounded rationality. Unlike Greif, however, who treats adoption of an institutional arrangement as a strategy in the game, Aoki sees institutions as a “by-product” – the rules or conditions that prescribe how the game is played. Aoki’s proposed methodology is comprehensive – it covers all kinds

of institutions, and recognizes the “embeddedness” and linkages in the evolution of institutions across domains (i.e., organizational, economic, social, and political spheres) and the strategic complementarities across these domains that are necessary for institutions to be enforceable. It also allows for possible multidirectional endogeneities in the evolution of such institutions. However, the comprehensiveness of the framework makes it difficult to obtain specific results.

Young offers a compromise between Greif and Aoki. As with Greif, the institution, or adoption thereof, is literally a strategy in the game, and not a rule governing the game. Thus, an institution becomes *de facto* established as a norm or “convention” as the strategy of adoption spreads among the population of players – that is, as more and more individuals literally adopt the institution. Young drops the classical-game framework and assumes that players are boundedly rational, but unlike Aoki, he proposes specific (evolutionary) games and dynamics to formally describe how institutions become established.

Our model is thus closest to Young’s in formulation and in theoretical results. While the specific dynamics we propose are different, we are able to show that with mutation, the strategy that survives in the long run is the risk-dominant strategy. This is generally consistent with Young and the evolutionary-game literature for 2×2 coordination games. Unlike Young, however, we stay closer to the NIE literature by distinguishing between formal rules and informal norms, by providing more intuition behind the mutation behavior of agents, and in interpreting the results. In Young’s model, the extent of bounded rationality is arbitrarily captured by a fixed probability of mutation, while in our model, the mutation rate can be affected by (random) changes in underlying conditions or the environment and can thus be time-varying and stochastic. Lastly, we also allow for instances in which agents are approximately rational and the initial play fully determines the long-run equilibrium. Consequently, our model is more comprehensive in that it recognizes cases in which risk dominance is important and cases in which it is not (and the initial play matters). This, in turn, has implications for the effectiveness of big-bang approaches to reform, which affect only the initial play, relative to gradualism, which can affect the more fundamental payoffs of the game.

6 Conclusions

While the RD is already established in the evolutionary-game literature, our proposed (stochastic) dynamic with Brownian drift and stochastic mutation rate is new. It is loosely based on Foster and Young, Fudenberg and Harris, and Cabrales, but we relate the mutation rate to the (stochastic) drift term. While there exist many other stochastic dynamics that capture mutation (and/or drift),¹⁷ our model supposes how the mutation rate could be not only time-varying, but also stochastic. To our best knowledge, apart from Robles and Houba and Tieman, results from evolutionary

¹⁷ See KMR, Young, Samuelson, Binmore, Samuelson and Vaughan, BLUME [2003], AMIR AND BERNINGHAUS [1996], FUDENBERG AND HARRIS [1992], Robson and Vega-Redondo, and VEGA-REDONDO [1996], [2003] for a survey.

games with time-varying, let alone stochastic, mutation rates are still not clearly generalizable.

Applied to the process of institutional change, our technique is closer to Young than to Aoki or Greif. Greif captures the notion of self-reinforcing equilibria, but his is still in the classical game-theoretic framework of hyper-rational players. Aoki allows for the bounded rationality of players, but he models institutions as a “by-product” of strategic interaction, and does not provide specific dynamics whose general properties can be obtained.

Intuitively, however, we draw more heavily from the NIE literature than Young in distinguishing between formal rules and informal or *de facto* norms, and in pointing out the implications of having boundedly rational players. Note that a critical observation in the NIE literature is that the problems inherent in reforming institutions emerge when there is a disjunction between formal rules and informal norms of behavior. The model we propose allows one to examine such cases of disjunction by considering how agents choose to comply with, or establish as informal norms, existing (formal) rules. Even when formal rules might initially converge into informal norms, random shocks (in the form of drift and mutation) could keep driving a wedge between them, which in turn might or might not lead back to convergence. The model thus provides an intuitively plausible model of the extent of path dependence or nonergodicity in institutional change.

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