

Public Goods, Corruption, and the Political Resource Curse

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Abstract

When do resource revenues increase corruption? I develop a model of public good provision by a politician who obtains rents by stealing government revenues or extracting bribes in exchange for public goods spending. I show there is a threshold level of revenues below which the politician does not steal, and therefore obtains rents only from bribes. Higher revenues unambiguously increase public goods spending, and decrease corruption (in the form of bribes) if the marginal social value of the public goods is sufficiently high. Above this threshold, revenues have no effect on spending, but unambiguously increases corruption (in the form of theft). Hence, a political resource curse emerges when resources provide ‘too much’ government revenues — that is, beyond a threshold level that corrupt politicians would credibly spend on public goods.

Keywords: corruption, public goods, theft of government revenues, bribery, political resource curse

JEL Codes: D73, H2, H41

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1 Introduction

Mounting empirical evidence suggests that government revenues from oil, natural resources, and other windfall gains increase corruption – see, e.g., Caselli and Michaels (2013), Brollo et al. (2013), Sala-i-Martin and Subramanian (2013), and Ferraz et al. (2012).¹ Moreover, as shown in Desierto (2018), formal models of the political resource curse posit that resource revenues provide rents which can be appropriated by corrupt public officials at the expense of public good provision. This mechanism reveals that corruption and public goods spending are intrinsically related.

Indeed, many papers demonstrate that public good provision can provide opportunities for corruption. Olken (2006, 2007), Olken and Pande (2012), Renikka and Svensson (2004), and Niehaus and Sukhtankar (2013) reveal sizeable leakages in the implementation of public programs and projects. Mauro (1998), Tanzi and Davoodi (1997, 2001), Gupta, Davoodi and Tiongson (2001), and Gupta, de Melo and Sharan (2001) suggest that corruption is associated with only some types of government expenditures – spending on military contracts and public works, in particular, are thought to generate large bribes and kickbacks. Arvate et al. (2010) and Hessami (2014), however, show that the positive association exists for most types of government expenditures, even across OECD countries.

If corruption and public spending are indeed related, such association should be more pronounced when the revenues that fund spending largely come from natural resources and similar windfall incomes. Yet even a cursory look at cross-country data suggests the opposite. Figure 1 shows that while, overall, the incidence of bribery increases with military spending, such association is only apparent for countries with little reliance on oil. In fact, for countries whose oil rents are greater than 10 percent of GDP, the association disappears.

Empirical and theoretical results on the political resource curse remain incongruous essentially because the relationship between corruption and public good provision is undertheorized. On the one hand, canonical models of the rent-seeking political agent – Barro (1973), Ferejohn (1986), Persson and Tabellini (2000), Bueno de Mesquita et al. (1999, 2003, 2010), which are applied to resource curse phenomena in Brollo et al. (2013), Abdih et al. (2012), Ahmed (2012), Smith (2008), and Robinson et al. (2006), show that public good provision is associated with less corruption. In these models, the agent can either spend government revenues on public goods, which benefit all citizens, or appropriate it for her own consumption and/or to buy political support. Corruption is tantamount to theft of government revenues. In this case, the agent is *revenue-seeking*. When the agent is revenue-seeking, public spending and corruption necessarily move in opposite directions, as more spending simply leaves less revenues for the agents' private use/consumption.

¹See also Tyburski (2014), Berdiev et al. (2013), Abdih et al. (2012), Ahmed (2012), Arezki and Bruckner (2011), Vicente (2010), Leite and Weidmann (1999), La Porta et al. (1999) For a survey, see Ross (2015).

On the other hand, the agent might want to increase spending in order to obtain rents — that is, she could be *expenditure-seeking*. This type of rent-seeking is captured in the common agency models of bribery, pioneered by Bernheim and Whinston (1986a, 1986b), Dixit, Grossman, and Helpman (1997), and Grossman and Helpman (1994, 2001), in which principals from the private sector offer bribes to their common political agent in exchange for their preferred policy, e.g. higher public-good spending.

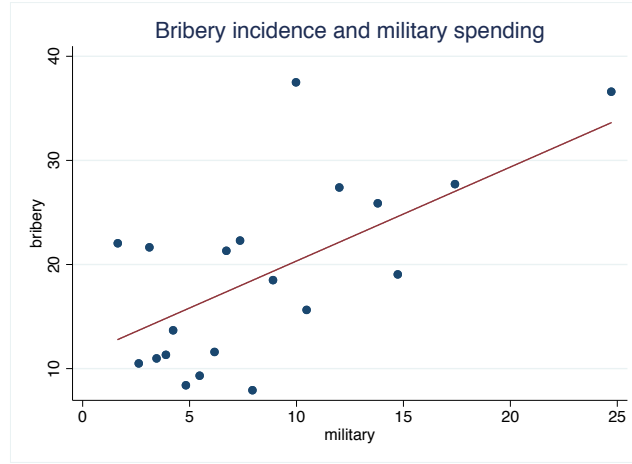
What is required is a model that allows for agents to be *both* revenue and expenditure-seeking. I provide what is—to the best of my knowledge—the first such model. The theoretical framework I propose not only generates novel insights into the relationship between public goods spending and corruption but, in so doing, also clarifies the conditions under which a political resource curse occurs.

I build on the work of Grossman and Helpman (2001) who apply the common agency model with complete information to the problem of the optimal allocation of government revenues between two sets of principals, one of which offers a menu of contributions or bribes in order to influence the agent to spend relatively more revenues towards that principal. Grossman and Helpman (2001) assume, however, that the agent spends all of the revenues and, thus, obtains rents only by receiving bribes. Since the bribes are given in exchange for spending, and more spending also increases the principals' utility, an increase in government revenues always induces higher total spending.

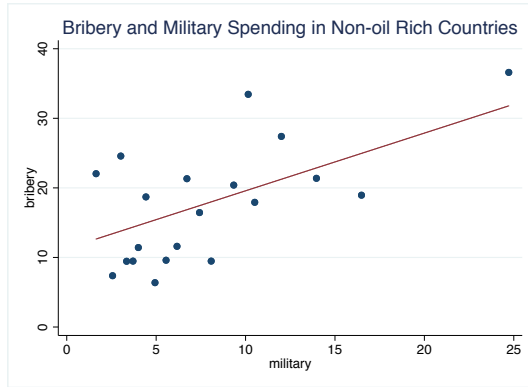
In my model, I allow for the possibility that the agent steals the revenues. In this case, the effect of increased revenues on spending is not obvious. The agent might spend all of the additional revenues in exchange for more bribes, but she might also want to keep them for herself. By incorporating theft into the model, I show that there is a threshold level of spending that the agent maintains. If revenues are at or below this threshold, the agent spends all of the revenues and, therefore, obtains rents only from bribes. In this case, increasing revenues up to the threshold unambiguously increases public spending. When revenues are larger than the threshold spending, the agent maintains the latter and steals all the extra revenues above the threshold. In equilibrium, any increase in revenues beyond the threshold has no effect on public spending, nor on bribes.

The intuition is that the bribing principal can obtain a higher share in the revenues only if those revenues are spent in the first place. Thus, unless the agent willingly spends the revenues, the bribe has to be sufficiently high so as to induce the agent to spend the revenues and to allocate more of it towards the bribing principal. However, the latter would not be willing to pay this much since inducing the agent to spend benefits *all* principals. In equilibrium, the amount of the bribe cannot prevent the theft of revenues – it can only pay for a higher share of the revenues that the agent is willing to spend. At some point, the agent will not want to keep increasing spending precisely because she can steal the revenues instead. The agent only needs to meet, at most, a threshold level

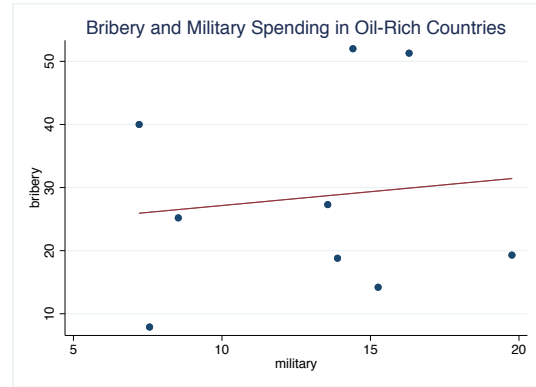
Figure 1: Does Corruption Increase with Military Spending?



(a) All available World Bank country-level data between 1997-2012



(b) Countries with oil rents less than 10% of GDP



(c) Countries with oil rents greater than 10% of GDP

This figure shows binned scatterplots of military spending and the incidence of bribery. Data used are from a pooled cross-section of countries for which some World Development Indicators are available between years 1997 to 2012 — specifically: *bribery*, which is the percentage of firms experiencing at least one bribe payment request; *military*, which is military expenditure as a percentage of GDP; and *oil*, which is oil rents as a percentage of GDP. Graph (a) uses all available data, while graphs (b) and (c) use subsets of the data for which oil rents are, respectively, less than and greater than 10% of GDP

of public spending, that is, without suffering the consequence of being removed from office. This is because the agent can use her rents from bribes and stolen revenues to gain political advantage by, say, swaying electoral outcomes.

The model thus has important implications for the political resource curse. Government revenues increase corruption at the expense of public good spending when the revenues are larger than some threshold. This suggests that countries that are heavily reliant on resource revenues are more likely to exceed the threshold, which enables corrupt politicians to engage in revenue-seeking, rather than expenditure-seeking, behavior. The reverse holds for countries that are less dependent on windfall

incomes. This would explain the seemingly paradoxical pattern shown in Figure 1. The lack of association between bribery and spending in oil-rent rich economies need not imply that there is no corruption, but that the rent-seeking is in the form of theft, rather than bribery.

While existing datasets on corruption do not distinguish between theft and bribery, some anecdotal evidence may render initial support to the model’s findings. Note, in particular, two of the biggest corruption scandals to date. In 2015, former Prime Minister Najib Razak was accused of stealing \$ 700 million from the government development company 1MDB. In 2014, public officials at Brazilian oil company Petrobras corporation were alleged to have taken \$ 350 million in bribes in exchange for awarding contracts to construction company Odebrecht. Both 1MDB and Petrobras are funded by oil revenues, but why did corruption occur in the form of theft in Malaysia and of bribery in Brazil? For Malaysia to have exceeded the threshold level of revenues that triggers theft, it must be that Malaysia’s economy is more dependent on resource revenues than Brazil’s. Indeed, Malaysia’s average income from natural resources over the period 1970-2016 is 17.64% of GDP, while Brazil’s is only 2.64%.²

The structure of the remainder of the paper is as follows. The next section formally derives results, analyzes the implications on social welfare, and interprets the political resource curse in the light of the results. In Section 3, I explicitly show that the revenue– and expenditure-seeking behavior of the agent occurs even when she can be made accountable to her principals through elections — such political accountability is imperfect because the rents from office can be used to influence electoral outcomes. Section 4 concludes with a summary of the contributions of the model.

2 The Model

The following game is by Grossman and Helpman (2001). Let T be government revenues that are to be spent on principal 1 and principal 2 by their common agent – a public official that has discretion over the use of T . Denote g_1 as the public good spending that the agent allocates to principal 1 and g_2 to principal 2. Principal 1 derives gross benefit $V(g_1)$, while principal 2 derives benefit $V(g_2)$, with $V'(\cdot) > 0, V''(\cdot) < 0$. Principal 1 offers the agent bribe b in exchange for g_1 . Its net benefit from public spending is thus $V(g_1) - b$. The agent then chooses an allocation (g_1, g_2) . It values rents, but also cares about social welfare.

In Grossman and Helpman, the only source of rents for the agent is the bribe payment. In contrast, I consider the possibility that the agent can also steal government revenues. Thus, let total rents R include both bribes and unspent revenues (which the government steals), i.e. $R =$

²See theglobeconomy.com. The 1MDB company was originally the Terengganu Investment Authority (TIA), which was funded by royalties and additional guarantees by the government based on future oil revenues. (See https://en.wikipedia.org/wiki/1Malaysia_Development_Berhad.)

$T - g_1 - g_2 + b$. The agent's utility is thus given by $U = \lambda[V(g_1) + V(g_2)] + (1 - \lambda)(T - g_1 - g_2 + b)$, where $\lambda \in (0, 1)$ is the weight it attaches to social welfare which, for now, is taken as given. Section 3 endogenizes it.

As standard in common agency models with complete information, the equilibrium spending allocation and bribe payment are jointly efficient for the agent and the principal who offers the bribe. That is, it is obtained by solving

$$\begin{aligned} & \max_{g_1, g_2, b} V(g_1) - b \\ \text{s.t. } & \lambda[V(g_1) + V(g_2)] + (1 - \lambda)(T - g_1 - g_2 + b) \geq \bar{U} \quad (a) \\ & g_1 + g_2 - T \leq 0 \quad (b), \end{aligned} \quad (1)$$

where \bar{U} is the agent's reservation utility - what it would obtain if it rejects principal 1's offer. Constraint (a) requires that the agent's utility when it accepts the bribe is at least as large as when it rejects it. The possibility of theft is captured by constraint (b) - if it is binding, i.e. $g_1 + g_2 = T$, then all revenues are spent and theft is not possible. If it is slack, then theft occurs, with the amount of stolen revenues equal to $T - g_1 - g_2$. I thus call constraint (b) the "no-theft constraint".

I first analyze the equilibrium in which the no-theft constraint binds and, thus, only bribery is the source of the agent's rents. I restrict the discussion to interior solutions.

2.1 Bribery

If the no-theft constraint binds, then $g_2 = T - g_1$ and problem (1) becomes³

$$\begin{aligned} & \max_{g_1, b} V(g_1) - b \\ \text{s.t. } & \lambda[V(g_1) + V(T - g_1)] + (1 - \lambda)(b) \geq \bar{U} \end{aligned} \quad (2)$$

In equilibrium, the above constraint binds with equality, which allows one to obtain the following expression for b :

$$b = \left(\frac{1}{1 - \lambda}\right) [\bar{U} - \lambda[V(g_1) + V(T - g_1)]], \quad (3)$$

which, when plugged into the maximand in (2), transforms (2) into the following unconstrained problem:

$$\max_{g_1} V(g_1) - \left(\frac{1}{1 - \lambda}\right) [\bar{U} - \lambda[V(g_1) + V(T - g_1)]]. \quad (4)$$

Equilibrium g_1^* thus satisfies the first-order condition (FOC) $F = V'(g_1^*) + \frac{\lambda}{1 - \lambda}V'(g_1^*) - \frac{\lambda}{1 - \lambda}V'(T - g_1^*) = 0$, or

$$V'(g_1^*) = \lambda V'(T - g_1^*). \quad (5)$$

³This is the exact same problem in Grossman and Helpman (2001).

That is, the equilibrium allocation attaches more weight to the marginal benefit from spending of the principal that offers a bribe, implying that $g_1^* > g_2^* = T - g_1^*$.

What are the effects of government revenues on public spending and corruption (in the form of bribe-rents)? First, Proposition 1 establishes that revenues increase both g_1^* and $g_2^* = T - g_1^*$ and, thus, total public spending. However, the increase in g_1^* may be smaller or larger than the increase in g_2^* . In particular, if the ratio of the rates of decrease of the marginal utilities from g_1^* and from g_2^* is larger (smaller) than the weight λ that the agent attaches to social welfare, then an increase in revenues induces a smaller (larger) increase in g_1^* relative to g_2^* . That is:

Proposition 1. *Let $x = \frac{V''(g_1^*)}{V''(T-g_1^*)}$. Then:*

$$\begin{aligned} (a) \text{ if } \lambda > x, \quad & \text{then } = \frac{dg_1^*}{dT} > \frac{dg_2^*}{dT} > 0. \\ (n) \text{ if } \lambda < x, \quad & \text{then } = \frac{dg_2^*}{dT} > \frac{dg_1^*}{dT} > 0. \\ (a) \text{ if } \lambda = x, \quad & \text{then } = \frac{dg_2^*}{dT} = \frac{dg_1^*}{dT} > 0. \end{aligned}$$

(All proofs are in appendix D.)

Next, to show the effect of revenues on bribe-rents, I conduct comparative statics on the equilibrium amount of bribes. The bribe equation (3) requires an expression for \bar{U} — which is the utility that the agent would obtain if she rejected principal 1's bribe offer. In this case, the agent's utility would be given by $\lambda[V(g_1) + V(T - g_1)]$, which she could maximize by choosing the first-best, socially optimal level of spending, i.e. g_1^0 . To see this, note that maximizing $\lambda[V(g_1) + V(T - g_1)]$ yields FOC $V'(g_1^0) = V'(T - g_1^0)$, which implies an equal allocation of T between sectors, that is, $g_1^0 = (T - g_1^0) = \frac{T}{2}$. Thus, if the agent rejects the bribe offer, she gets $\bar{U} = \lambda[V(\frac{T}{2}) + V(\frac{T}{2})] = 2\lambda V(\frac{T}{2})$ which, when plugged into equation (3) gives the equilibrium amount of bribes:

$$b^* = \frac{\lambda}{1 - \lambda} \left[2V\left(\frac{T}{2}\right) - V(g_1^*) - V(T - g_1^*) \right]. \quad (6)$$

Thus, in equilibrium, the bribe compensates the agent for a fraction $\frac{\lambda}{1-\lambda}$ of the loss in social welfare.

Differentiating (6) with respect to T reveals that government revenues have an ambiguous effect on bribe-rents. Specifically:

Proposition 2. *Government revenues may increase or decrease corruption. Specifically, let $y = \frac{V'(\frac{T}{2}) - V'(T-g_1^*)}{V'(g_1^*) - V'(T-g_1^*)}$. Then:*

$$\begin{aligned} (a) \text{ if } \frac{dg_1^*}{dT} < y, \quad & \text{then } = \frac{\partial b^*}{\partial T} > 0 \\ (b) \text{ if } \frac{dg_1^*}{dT} > y, \quad & \text{then } = \frac{\partial b^*}{\partial T} < 0. \\ (c) \text{ if } \frac{dg_1^*}{dT} = y, \quad & \text{then } = \frac{\partial b^*}{\partial T} = 0. \end{aligned}$$

That is, if revenues increase, principal 1 will want to increase (decrease) the amount of the bribe if the agent will want to allocate the additional revenues towards principal 1 at a rate that is below (above) some threshold y . In turn, this threshold captures the marginal value of the spending on principal 1, relative to the spending on principal 2 (see the denominator of y). Note that when the former is much larger than the latter, the threshold is smaller, which makes a decrease in corruption (case (b)) more likely.

In an equilibrium in which the no-theft constraint binds, government revenues do not generate a political resource curse. Revenues decrease corruption if the marginal value of the public spending from which bribes are extracted is sufficiently high. Yet, even when this marginal value is low and corruption increases, total public good spending unambiguously increases at the rate at which revenues increase.

The following examples use various functional forms for the principals' utility from public spending.

Running Example 1. Suppose $V(g_i) = \ln g_i$. Then $g_1^* = \frac{T}{1+\lambda}$, $g_2^* = \frac{\lambda T}{1+\lambda}$, and $\frac{dg_1^*}{dT} = \frac{1}{1+\lambda} > \frac{\lambda}{1+\lambda} = \frac{dg_2^*}{dT}$. Condition (a) of Proposition 1 is satisfied for all $\lambda \in (0, 1)$, since x in this case is equal to λ^2 . It can also be shown that $b^* = \frac{\lambda}{1-\lambda}[2\ln(\frac{T}{2}) - \ln(\frac{T}{1+\lambda}) - \ln(\frac{\lambda T}{1+\lambda})]$ and, thus, $\frac{\partial b^*}{\partial T} = 0$. Condition (c) of Proposition 2 is satisfied for all $\lambda \in (0, 1)$, since y in this case is equal to $\frac{1}{1+\lambda}$.

Running Example 2. Suppose $V(g_i) = \sqrt{g_i}$. Then $g_1^* = \frac{T}{1+\lambda^2}$, $g_2^* = \frac{\lambda^2 T}{1+\lambda^2}$, and $\frac{dg_1^*}{dT} = \frac{1}{1+\lambda^2} > \frac{\lambda^2}{1+\lambda^2} = \frac{dg_2^*}{dT}$. Condition (a) of Proposition 1 is satisfied for all $\lambda \in (0, 1)$, since x in this case is equal to λ^3 . As for the equilibrium bribes, it can be shown that $b^* = \frac{\lambda}{1-\lambda}[2\sqrt{\frac{T}{2}} - \sqrt{\frac{T}{1+\lambda^2}} - \sqrt{\frac{\lambda^2 T}{1+\lambda^2}}]$, and that $\frac{\partial b^*}{\partial T} = \frac{\lambda}{1-\lambda}[\frac{1}{\sqrt{2T}} - \frac{(1+\lambda)}{2\sqrt{(1+\lambda^2)+T}}]$. Thus, it is now the case that $\frac{\partial b^*}{\partial T} < 0$, since $\frac{1}{\sqrt{2T}} < \frac{(1+\lambda)}{2\sqrt{(1+\lambda^2)+T}}$ or, simplifying, $\lambda < 1$. Condition (b) of Proposition 2 is satisfied for all $\lambda \in (0, 1)$, since in this case, $\frac{dg_1^*}{dT} = \frac{1}{1+\lambda^2} > \frac{(1-\lambda)\sqrt{2}}{\sqrt{1+\lambda^2}} - \frac{1}{\lambda} - 1 = y$. To see this, one can simplify the latter inequality to $\frac{1}{\lambda} > (\sqrt{2(1+\lambda^2)} - \lambda)(1-\lambda)$ and note that the LHS is greater than 1, while the RHS is less than 1 for all $\lambda \in (0, 1)$.

Running Example 3. Suppose $V(g_i) = -\frac{1}{g_i}$. Then $g_1^* = \frac{T}{1+\sqrt{\lambda}}$, $g_2^* = \frac{\sqrt{\lambda}T}{1+\sqrt{\lambda}}$, and $\frac{dg_1^*}{dT} = \frac{1}{1+\sqrt{\lambda}} > \frac{\sqrt{\lambda}}{1+\sqrt{\lambda}} = \frac{dg_2^*}{dT}$. Condition (a) of Proposition 1 is satisfied for all $\lambda \in (0, 1)$, since x in this case is equal to $\lambda\sqrt{\lambda}$. Equilibrium bribe is $b^* = \frac{(1+\lambda)\sqrt{\lambda}-2\lambda}{(1-\lambda)T}$. Thus, in this case, $\frac{\partial b^*}{\partial T} = \frac{2\lambda-(1+\lambda)\sqrt{\lambda}}{(1-\lambda)T^2} > 0$, since $2\lambda > (1+\lambda)\sqrt{\lambda}$ for all $\lambda \in (0, 1)$.⁴ Condition (a) of Proposition 2 is satisfied since $\frac{dg_1^*}{dT} = \frac{1}{1+\sqrt{\lambda}} < \frac{4\lambda-(1+\sqrt{\lambda})^2}{(\lambda-1)(1+\sqrt{\lambda})^2} = y$, which simplifies to $2\lambda > (1+\lambda)\sqrt{\lambda}$.

⁴To see this, note that $2 > 1 + \lambda$ and $\lambda > \sqrt{\lambda}$.

2.2 Bribery and Theft

I now consider the case when the no-theft constraint is non-binding/slack, which implies that theft is now also possible. Again, I restrict the analysis to interior solutions. Recall problem (1), in which $g_2 \neq T - g_1$:

$$\begin{aligned} & \max_{g_1, g_2, b} V(g_1) - b \\ \text{s.t. } & \lambda[V(g_1) + V(g_2)] + (1 - \lambda)(T - g_1 - g_2 + b) \geq \bar{U} \quad (a) \\ & g_1 + g_2 - T \leq 0 \quad (b), \end{aligned} \quad (7)$$

and where (b) is the no-theft constraint. With constraint (a) holding with equality in equilibrium, the problem can be simplified into

$$\begin{aligned} & \max_{g_1, g_2} V(g_1) - \frac{1}{1 - \lambda} [\bar{U} - \lambda[V(g_1) + V(g_2)]] + T - g_1 - g_2 \\ \text{s.t. } & g_1 + g_2 - T \leq 0 \end{aligned} \quad (8)$$

To obtain the equilibrium allocation and total rents when both bribery and theft can occur, one needs to solve (8) for the case when the no-theft constraint is slack. In this case, the necessary conditions for optimal g_1^*, g_2^*, γ^* are given by the following Kuhn-Tucker conditions:

$$V'(g_1^*) + \frac{\lambda}{1 - \lambda} V'(g_1^*) - 1 - \gamma^* = 0 \quad (9)$$

$$\frac{\lambda}{1 - \lambda} V'(g_2^*) - 1 - \gamma^* = 0 \quad (10)$$

$$\gamma^*(g_1^* + g_2^* - T) = 0, \quad (11)$$

where γ is the Lagrange multiplier — the ‘shadow price’ of preventing theft.

The following results show that not all revenues are stolen, and that some amount of spending is allocated to both principals. However, beyond this minimum spending, additional revenues have no effect on spending since they are all stolen. Finally, I compare the effect of revenues on rents when the only source is bribery with the effect when both theft and bribery can occur. I find an ambiguous effect — an increase in revenues may induce lower or higher rents from the former than from the latter.

To proceed, Proposition 3 first establishes that there is some minimum amount of revenues that are not stolen but are instead spent on both principals 1 and 2.

Proposition 3. *Even if theft occurs in equilibrium, some public spending are still allocated, i.e. $g_1^*, g_2^* > 0$.*

This implies that at and below this threshold level of revenues, the no-theft constraint binds, in which case bribes, as the only source of rents, can increase or decrease with revenues, as shown in Section 2.

To be able to conduct comparative statics on bribes, total rents, and public spending at revenues above the threshold level, I first solve for equilibrium bribe b^* when the no-theft constraint is non-binding. Constraint (a) in (7) implies that $b^* = \frac{1}{1-\lambda}[\bar{U} - \lambda[V(g_1^*) + V(g_2^*)]] + T - g_1^* - g_2^*$. To get the agent's reservation utility \bar{U} , note that if the agent rejects the bribe offer, she will obtain utility from social welfare and stolen revenues, which she can maximize by choosing g_1^0, g_2^0 via the following optimization problem:

$$\begin{aligned} \max_{g_1, g_2} \lambda[V(g_1) + V(g_2)] + (1 - \lambda)(T - g_1 - g_2) \\ \text{s.t. } g_1 + g_2 - T \leq 0 \end{aligned} \quad (12)$$

Necessary for g_1^0, g_2^0, γ^0 are the following Kuhn-Tucker conditions:

$$\lambda V'(g_1^0) - (1 - \lambda) - \gamma^0 = 0 \quad (13)$$

$$\lambda V'(g_2^0) - (1 - \lambda) - \gamma^0 = 0 \quad (14)$$

$$\gamma^0(g_1^0 + g_2^0 - T) = 0 \quad (15)$$

A result similar to Proposition 3 establishes that $g_1^0, g_2^0 > 0$ (see appendix D). Thus, if the agent rejects the bribe offer, she obtains utility $\bar{U} = \lambda[V(g_1^0) + V(g_2^0)] + (1 - \lambda)(T - g_1^0 - g_2^0)$ which, when plugged into the expression for b^* , gives the equilibrium amount of bribes at revenue levels above the threshold:

$$b^* = \frac{\lambda}{1 - \lambda}[V(g_1^0) + V(g_2^0) - V(g_1^*) - V(g_2^*)] - (g_1^0 + g_2^0 - g_1^* - g_2^*). \quad (16)$$

The following comparative static results demonstrate that revenues above the threshold have no effect on spending, nor on bribes.

Proposition 4. *Government revenues have no effect on g_1^* or g_2^* , i.e. $\frac{dg_1^*}{dT}, \frac{dg_2^*}{dT} = 0$.*

Lemma 1. *Government revenues have no effect on g_1^0 or g_2^0 , i.e. $\frac{dg_1^0}{dT} = 0$ and $\frac{dg_2^0}{dT} = 0$.*

Proposition 5. *Government revenues have no effect on the equilibrium bribe, i.e. $\frac{\partial b^*}{\partial T} = 0$.*

The following examples revisit the various functional forms for principals' utility in section 2.

Running Example 1. When the no-theft constraint is non-binding, $\gamma^* = 0$ and, thus, from equation (9), $g_1^* = \frac{1}{1-\lambda}$, and from equation (10), $g_2^* = \frac{\lambda}{1-\lambda}$. Thus, $\frac{dg_1^*}{dT} = \frac{dg_2^*}{dT} = 0$, which is consistent with Proposition 4. It can also be shown that $b^* = \frac{\lambda}{1-\lambda}[\ln(\frac{\lambda}{1-\lambda}) - \ln(\frac{1}{1-\lambda})] + 1$ and, thus, $\frac{\partial b^*}{\partial T} = 0$, which is consistent with Proposition 5.

Running Example 2. In this case, $g_1^* = \frac{1}{4(1-\lambda)^2}$, $g_2^* = \frac{\lambda^2}{4(1-\lambda)^2}$, and $\frac{dg_1^*}{dT} = \frac{dg_2^*}{dT} = 0$, which is consistent with Proposition 4. Equilibrium bribe is $b^* = \frac{\lambda}{1-\lambda}[\frac{1}{\lambda-1} - \frac{1+\lambda}{2(1-\lambda)}] - \frac{1}{2(\lambda-1)^2} + \frac{1+\lambda^2}{4(1-\lambda)^2}$ and, thus, $\frac{\partial b^*}{\partial T} = 0$, which is consistent with Proposition 5.

Running Example 3. In this case, $g_1^* = \frac{1}{\sqrt{1-\lambda}}$, $g_2^* = \frac{\sqrt{\lambda}}{\sqrt{1-\lambda}}$, and $\frac{dg_1^*}{dT} = \frac{dg_2^*}{dT} = 0$, which is consistent with Proposition 4. Equilibrium bribe is $b^* = \frac{\lambda}{1-\lambda}[\sqrt{1-\lambda} - \frac{\sqrt{1-\lambda}}{\sqrt{\lambda}}] + \frac{1-\sqrt{\lambda}}{\sqrt{1-\lambda}}$ and, thus, $\frac{\partial b^*}{\partial T} = 0$, which is consistent with Proposition 5.

Note that if public spending does not change, then additional government revenues above the threshold level are all stolen, implying that theft increases at a rate of 1. Furthermore, because the amount of bribes is also fixed, total rents grow at the rate of growth of theft. That is, above the threshold level of revenues:

Corollary 1. *Any additional government revenues are stolen, and $\frac{\partial R^*}{\partial T} = 1$.*

Finally, what are the relative magnitudes of the effect of government revenues on corruption above and below the threshold? It turns out that revenues that are above the threshold need not always generate rents at a higher rate than revenues below the threshold do. If the marginal value of the public spending from which bribes are extracted is sufficiently low, then, at revenues below the threshold, not only do bribes increase with revenues — recall Proposition 2, but the rate of increase can be higher than the rate at which total rents increase above the threshold, i.e. when both theft and bribes are the source of rents. That is, case (a) in Proposition 6 below is more likely to hold.

Proposition 6. *Denote total corruption when theft does not occur as $R^{\mathcal{F}}$ and R^* when theft occurs, in equilibrium, and re-label equilibrium g_1^* and g_2^* obtained in the case of no theft as $g_1^{\mathcal{F}}$ and $g_2^{\mathcal{F}}$. Let $z = \frac{V'(\frac{T}{2}) - V'(T - g_1^{\mathcal{F}}) - (\frac{1-\lambda}{\lambda})}{V'(g_1^{\mathcal{F}}) - V'(T - g_1^{\mathcal{F}})}$. Then:*

- (a) if $\frac{dg_1^{\mathcal{F}}}{dT} < z$, then $\frac{\partial R^{\mathcal{F}}}{\partial T} > \frac{\partial R^*}{\partial T}$;
- (b) if $\frac{dg_1^{\mathcal{F}}}{dT} > z$, then $\frac{\partial R^{\mathcal{F}}}{\partial T} < \frac{\partial R^*}{\partial T}$;
- (c) if $\frac{dg_1^{\mathcal{F}}}{dT} = z$, then $\frac{\partial R^{\mathcal{F}}}{\partial T} = \frac{\partial R^*}{\partial T}$.

The following demonstrate cases (a), (b), and (c) using the same functional forms in previous examples.

Running Example 1. Recall from Section 2 that $\frac{\partial b^*}{\partial T} = \frac{\partial R^{\mathcal{F}}}{\partial T} = 0$ when $V(g_i) = \ln g_i$, and is thus less than $\frac{\partial R^*}{\partial T} = 1$. This is consistent with condition (b) of Proposition 6, since in this case, $z = \frac{1}{1+\lambda} + \frac{T}{\lambda-1}$, which is less than $\frac{dg_1^{\mathcal{F}}}{dT} = \frac{1}{1+\lambda}$.

Running Example 2. Recall from Section 2 that $\frac{\partial b^*}{\partial T} < 0$. Thus, $\frac{\partial R^F}{\partial T} < \frac{\partial R^*}{\partial T}$. This is consistent with condition (b) of Proposition 6 - note that $\frac{dg_1^F}{dT} = \frac{1}{1+\lambda^2} > \frac{\lambda\sqrt{2}-\sqrt{\lambda^2+1}-2\sqrt{T}(1-\lambda)}{\sqrt{\lambda^2+1}(\lambda-1)} = z$. To see this, note that the inequality can be reduced to $\frac{2T(1+\lambda^2)+\sqrt{\lambda^2+1}}{1+\lambda^2} > \frac{\lambda\sqrt{2}-\sqrt{\lambda^2+1}}{\lambda-1}$, and it can be shown that the LHS of this inequality is greater than 1, while the RHS is less than 1.

Running Example 3. Recall from Section 2 that $\frac{\partial b^*}{\partial T} = \frac{2\lambda-(\lambda+1)\sqrt{\lambda}}{(1-\lambda)T^2} > 0$. Note that $\frac{2\lambda-(\lambda+1)\sqrt{\lambda}}{(1-\lambda)T^2} < 1$ if, simplifying, $\lambda < \frac{T^2+\sqrt{\lambda}}{T^2+2-\sqrt{\lambda}} = \bar{\lambda}$. Thus, $\frac{\partial R^F}{\partial T} < \frac{\partial R^*}{\partial T}$ if $\lambda < \bar{\lambda}$, while $\frac{\partial R^F}{\partial T} \geq \frac{\partial R^*}{\partial T}$ if $\lambda \geq \bar{\lambda}$. These are consistent with the conditions of Proposition 6 - for instance, one can set $\frac{dg_1^F}{dT} = \frac{1}{1+\sqrt{\lambda}} > \frac{4\lambda-(1+\sqrt{\lambda})^2-T^2(1-\lambda)}{(1+\sqrt{\lambda})^2(\lambda-1)} = z$ to capture condition (b), which precisely reduces to $\lambda < \frac{T^2+\sqrt{\lambda}}{T^2+2-\sqrt{\lambda}} = \bar{\lambda}$. Conditions (a) and (c) easily follow.

Summarizing these results, Figure 2 depicts the effect of government revenues on public-good spending and corruption. Propositions 3 and 4 imply that there is a threshold amount of revenues \bar{T} at which total spending is positive, but beyond which spending does not increase further. Below the threshold, the no-theft constraint binds and, thus, all revenues are spent. Total public-good spending $S = g_1 + g_2$ thus has slope equal to 1 below \bar{T} and 0 thereafter. When the no-theft constraint binds, Proposition 2 shows that bribes may increase or decrease. Consider, then, bribe curves b_1 and b_2 which show different possibilities below \bar{T} but which, beyond \bar{T} , have slope 0 as implied by Proposition 5. Lastly, total rents below \bar{T} come solely from bribes, in which case R_1 and R_2 coincide with b_1 and b_2 , respectively. Above \bar{T} , rents come from both bribes and theft, with the amount of bribes fixed at b_1 and b_2 , and all additional revenues are stolen. Thus, above \bar{T} , R_1 and R_2 have as their y -intercepts their respective intersections with b_1 and b_2 , and slope equal to 1.

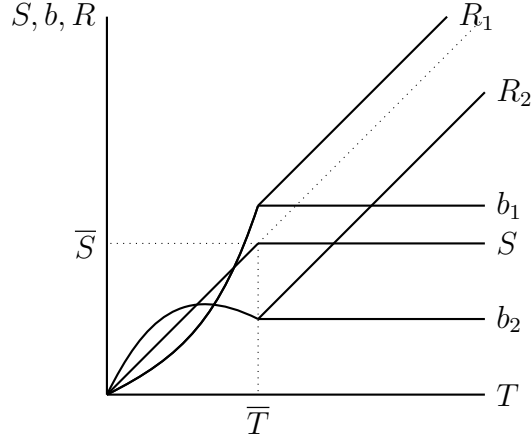
2.3 Social Welfare Loss from Theft and Bribery

The two sources of corruption in the model — bribery and theft, produce two kinds of social welfare losses. Bribery induces an inter-sectoral misallocation of total revenues since it buys the bribing principal a higher share in total revenues. Meanwhile, theft is an underspending of revenues.

In an equilibrium in which the no-theft constraint binds, the only source of corruption is bribery, which implies that the only type of social inefficiency is the misallocation of revenues between the principals/sectors. In contrast, in an equilibrium in which the no-theft constraint is slack, both bribery and theft occur, but where bribery is limited to some threshold, above which all additional rents come from theft. Thus, in this case, there are welfare losses both from the misallocation and underspending of revenues.

One could also compare the losses from such equilibria to the off-equilibrium cases in which the

Figure 2: Effect of Revenues on Public Spending and Corruption



This figure depicts the effect of revenues on public spending and corruption. There is a threshold level of revenues \bar{T} below which theft is not possible, in which case all revenues are spent and the only source of rents are bribes which may increase or decrease with revenues. Above \bar{T} , all additional revenues are stolen, public spending remains constant, and total rents come from a fixed amount of bribes and stolen revenues that are increasing at the same rate as revenues.

agent rejects the bribe, in which case no bribery occurs. The following four cases thus exhaust the different scenarios:

Case 1: Only Bribery, No Theft

When theft is not possible, but the government accepts bribes, there is no underspending of revenues, but a misallocation thereof, in which the principal/sector that bribes receives a higher share. Recall (from equation (5)) that the equilibrium allocation is given by the FOC: $V'(g_1^*) = \lambda V'(T - g_1^*)$, where $\lambda < 1$, which implies that $g_1^* > g_2^* = T - g_1^*$. Thus, even though principals derive the same marginal utility from public spending, principal 1 obtains a larger share. Note, however, that because $\lambda > 0$, the condition is *not* $V'(g_1^*) = 0$, which implies $g_1^* < T$. Otherwise, if λ were zero, problem (2) would yield FOC $V'(g_1^*) = 0$, which implies $g_1 = T$. Because the government also cares about social welfare, some spending also has to be allocated to the non-bribing sector. Thus, the bribe compensates the government for the loss in social welfare from ‘over-allocating’ to the bribing sector. In effect, the inefficiency in intersectional allocation is mitigated because $\lambda > 0$.

Case 2: No Bribery, No Theft

If the government were to reject the bribe offer, recall that it would get its reservation utility $\lambda[V(g_1) + V(T - g_1)]$ (in which bribes are zero). It would then choose spending to maximize this utility, which would yield FOC $V'(g_1^0) = V'(T - g_1^0)$, implying that $g_1^0 = g_2^0 = T - g_1^0 = \frac{T}{2}$. That is, the agent would allocate revenues exactly according to the principals’ marginal utilities from

spending. Thus, if there were no corruption - no theft or bribery, inter-sectoral allocation is socially efficient. There is also no welfare loss from underspending, since $g_1^0 + g_2^0 = T$.

Case 3: Bribery and Theft

If theft were now possible, and the government also accepts bribes, then the Kuhn-Tucker conditions given by equations (9) to (11) imply that the allocation is such that $V'(g_1^*) = \lambda V'(g_2^*)$, where $g_1^* + g_2^* < T$. Again, there is intersectoral misallocation because $\lambda < 1$, but it is mitigated because λ is not equal to zero. In addition, there is also underspending of total revenues since $g_1^* + g_2^* < T$.

Case 4: No Bribery, Only Theft

If the government were to reject the bribes when theft is possible, its reservation utility includes the rents she would derive from stolen revenues. In this case, the Kuhn-Tucker conditions given by equations (13) to (15) imply that the the government would choose an allocation such that $V'(g_1^0) = V'(g_2^0)$, where $g_1^0 + g_2^0 < T$. Inter-sectoral allocation is now socially efficient, but there is underspending of revenues.

Case 1 shows that the equilibrium when the no-theft constraint is binding generates welfare loss only from the misallocation of revenues, while case 3 shows that the equilibrium when the no-theft constraint is slack produces losses from both the misallocation and underspending of revenues. Since the no-theft constraint is slack when actual revenues are above some threshold, one may be tempted to infer that higher revenues always generate higher losses (as there would be two sources of welfare losses once revenues are above the threshold). However, this is not the case. I show in appendix A that the amount of bribes when revenues are below the threshold may be sufficiently large, and the bribes when revenues are above the threshold sufficiently small, such that the welfare-loss differential from bribes may be greater than the differential from theft. As a result, the social welfare loss (from bribes) at revenues below the threshold may be larger than the losses (from bribes and theft) at revenues above the threshold. Thus, it is not always true that losses are larger at higher levels of revenues.⁵

Why is it that only cases 1 and 3 can be supported in any equilibrium of the model? If we allow for the possibility of bribery, the bribing principal can now match the agent's reservation utility — what she would get if she rejected the bribe. That is, if the principal and agent can freely negotiate the amount of bribes, there is an amount that can induce the agent to accept the bribe. Thus, cases 2 and 4 in which bribes are rejected are off any equilibrium path. Note, however, that the extent to which the bribe can match the agent's reservation utility is limited. When revenues increase beyond some threshold, all additional rents now come from theft. This is because if the option to steal revenues now becomes viable, any additional bribe would not only have to compensate the agent for

⁵Thus, notice from Figure 2 that there is a region below \bar{T} (i.e. from the point of intersection between b_1 and b_2 until \bar{T}) in which bribes b_1 are larger than total rents from theft and bribes R_2 in a region above \bar{T} (i.e. from \bar{T} until the intersection of b_1 and R_2).

the loss in social welfare from the misallocation of revenues, but would also need to compensate the agent for foregoing the theft of revenues. This means that beyond the threshold, the (principal's) marginal cost of preventing theft is larger than the (agent's) marginal utility from theft. Thus, all revenues beyond the threshold are stolen.

While the *existence* of such a threshold is implied by the results, the value of the threshold cannot be determined in the model. One can solve for the equilibrium when the no-theft constraint binds, and when it is slack, but there is nothing that determines *ex ante* whether the constraint is binding or slack. One could indeed solve for \bar{g}_1, \bar{g}_2 when the constraint is binding, and let its sum $\bar{g}_1 + \bar{g}_2 = \bar{T}$ be the threshold amount of revenues above which theft is possible. That is, there is some minimum demand for spending that has to be met, and only when it is met can the agent steal the 'extra' revenues. Notice, however, that to obtain threshold \bar{T} , one has to assume that the constraint is binding. Thus, the model cannot solve for the value of \bar{T} .

2.4 An Application to the Political Resource Curse

The model demonstrates that the nature of the relationship between corruption and public-good spending depends on the existence of some threshold level of revenues. For an economy with government revenues above the threshold, the agent can keep public spending constant even while revenues are increasing. This implies that she can do so while remaining in office. In Section 3, I propose a model that can sustain this equilibrium — one in which the corrupt agent survives electoral competition by using her rents to buy votes. By such mechanism, the amount of public spending associated with the threshold level of revenues captures, as it were, the maximum value of public goods that is credibly demanded by the electorate. Beyond that level, it must be that their marginal utility from directly sharing in the agent's rents, i.e. from selling their votes, is greater than that from obtaining additional public goods.

When might government revenues exceed the threshold? I conjecture that an economy is more likely to be above the threshold the greater its reliance on revenues from oil, natural resources, and other windfall gains. A large influx of windfall income that flow directly to public coffers might be more easily captured through direct appropriation rather than indirectly by spending the windfall on public goods and extracting bribes therefrom. Thus, in equilibrium, a corrupt agent overseeing an economy with considerable windfall revenues would be more likely to increase her rents as revenues increase by engaging in more theft, rather than more bribery.⁶ In contrast, an economy that relies more heavily on tax revenues would be more likely to be below the threshold,

⁶This is indeed consistent with the formal literature on the political resource curse, where rent-seeking is modeled as theft or the appropriation of resource revenues (see Desierto (2018)). The results here thus provide explicit justification for why corruption is more aptly modeled as theft, rather than bribe-taking, when depicting a political resource curse.

in which case the agent can only keep extracting (bribe-)rents by spending more.

To show indirectly that such conjecture is plausible, Figure 3 plots military spending and corruption among countries that are reliant on oil revenues — those with oil rents greater than 10 percent of GDP. In lieu of a measure for the theft of revenues, for which data are unavailable, panel (a) uses (the incidence of) bribery, while panel (b) proxies for bribery by using a measure of corruption that describes the lack of transparency in the public sector. There appears to be no association between military spending and bribery, which would be consistent with the model since the latter predicts that an economy above the threshold level of revenues would remain at a fixed level of public spending and, thus, of bribe-rents. That is, if, above the threshold, the government increases rents by stealing more (and keeping spending fixed) rather than taking more bribes from higher spending, then spending and bribery would be unrelated. Note that the same pattern roughly holds when the extent of reliance on oil is increased to greater than 20 percent of GDP (panels (c) and (d).) In contrast, Figure 4 plots military spending and bribery among countries with less reliance on oil, and shows that the two variables increase together. If such countries were indeed below the threshold (and the marginal value of the public goods from which bribes are extracted is sufficiently low), the model predicts that both bribe-rents and public spending would increase with revenues, and would thus be positively associated.

Figure 5 confirms the patterns for countries that are more, and less, reliant on other types of windfall income, e.g. revenues from minerals and foreign aid.

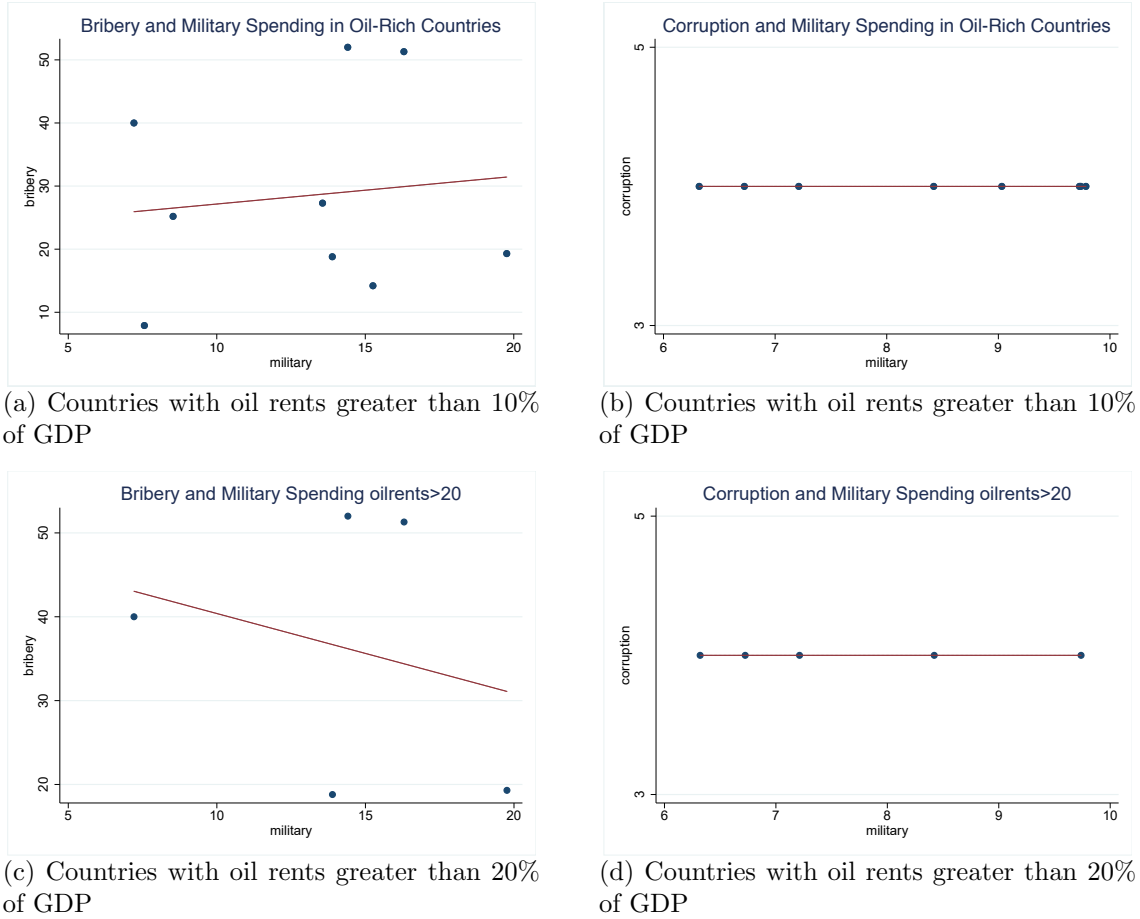
The model can thus be used to explain the political resource curse. An increase in government revenues unambiguously increases corruption (in the form of theft) at the expense of public goods — that is, the political resource curse always exists, when revenues exceed the minimum amount of public spending that (just) satisfies the credible level of demand for public goods.

3 Political Accountability

Thus far, it is assumed that the government values social welfare to some extent $\lambda \in (0, 1)$. This variable can capture institutional checks and balances that limit the extent of rent-seeking by the government. However, it can also describe the extent of political competition which pressures the government to be more accountable to citizens. In this case, that $\lambda < 1$ implies that political accountability is imperfect. I now show that this is supportable if candidates in elections share their rents with the electorate by buying votes.⁷ That is, I endogenize λ by modeling electoral

⁷This is only one example. One can envisage situations in which politicians share rents through other forms of patronage. Neither is electoral competition the only mechanism of (imperfect) political accountability that could support a rent-seeking equilibrium — a similar logic can operate under alternative forms of competition, e.g. via selectorate models in which the politician forms a coalition of supporters by offering to transfer some of her rents to

Figure 3: Bribery and Spending Unrelated Above the Threshold

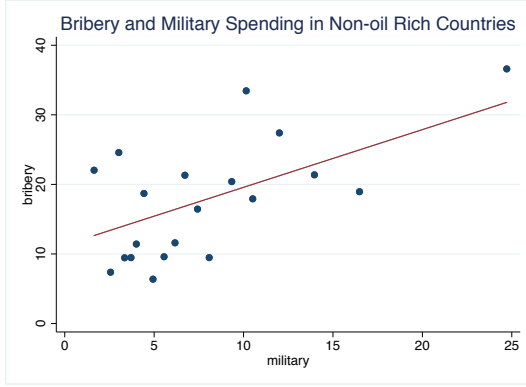


This figure shows binned scatterplots of military spending and two alternative measures of corruption, for countries that are more reliant on oil. Data used are from a pooled cross-section of countries for which some World Development Indicators are available between years 1997 to 2012 — specifically: *bribery*, which is the percentage of firms experiencing at least one bribe payment request; *corruption*, which is the CPIA transparency, accountability, and corruption in the public sector rating (with 1 re-coded as most, and 6 least, transparent); *military*, which is military expenditure as a percentage of GDP; and *oil*, which is oil rents as a percentage of GDP

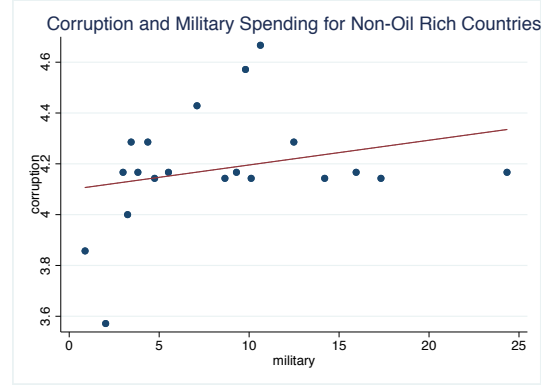
competition.

I now consider two candidates competing in elections, and let principal 1 offer bribes to each of them in exchange for a higher share in spending allocations once the candidate is in office. Each candidate then uses their respective bribes to buy votes. In equilibrium, the candidate that is more likely to win would allocate less spending to the bribing principal, which induces the latter to offer a bribe amount that is larger than what it offers to the other candidate. That is, campaign money (bribes) follows the more advantaged, i.e. popular, candidate. However, when candidates can also use stolen revenues as additional funds, the more popular candidate does not necessarily obtain a coalition members.

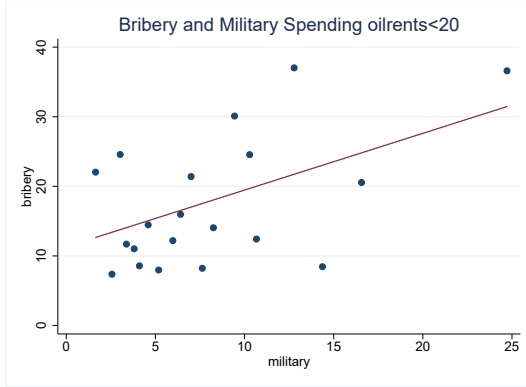
Figure 4: Bribery and Spending Positively Associated Below the Threshold



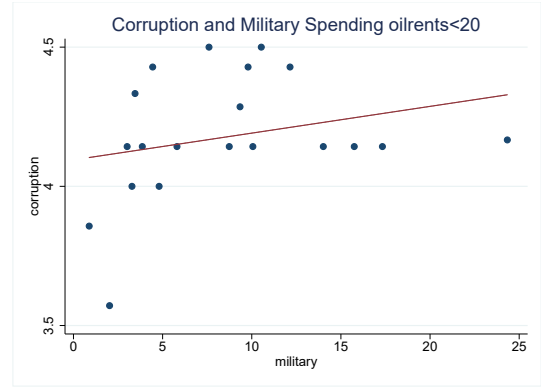
(a) Countries with oil rents less than 10% of GDP



(b) Countries with oil rents less than 10% of GDP



(c) Countries with oil rents less than 20% of GDP



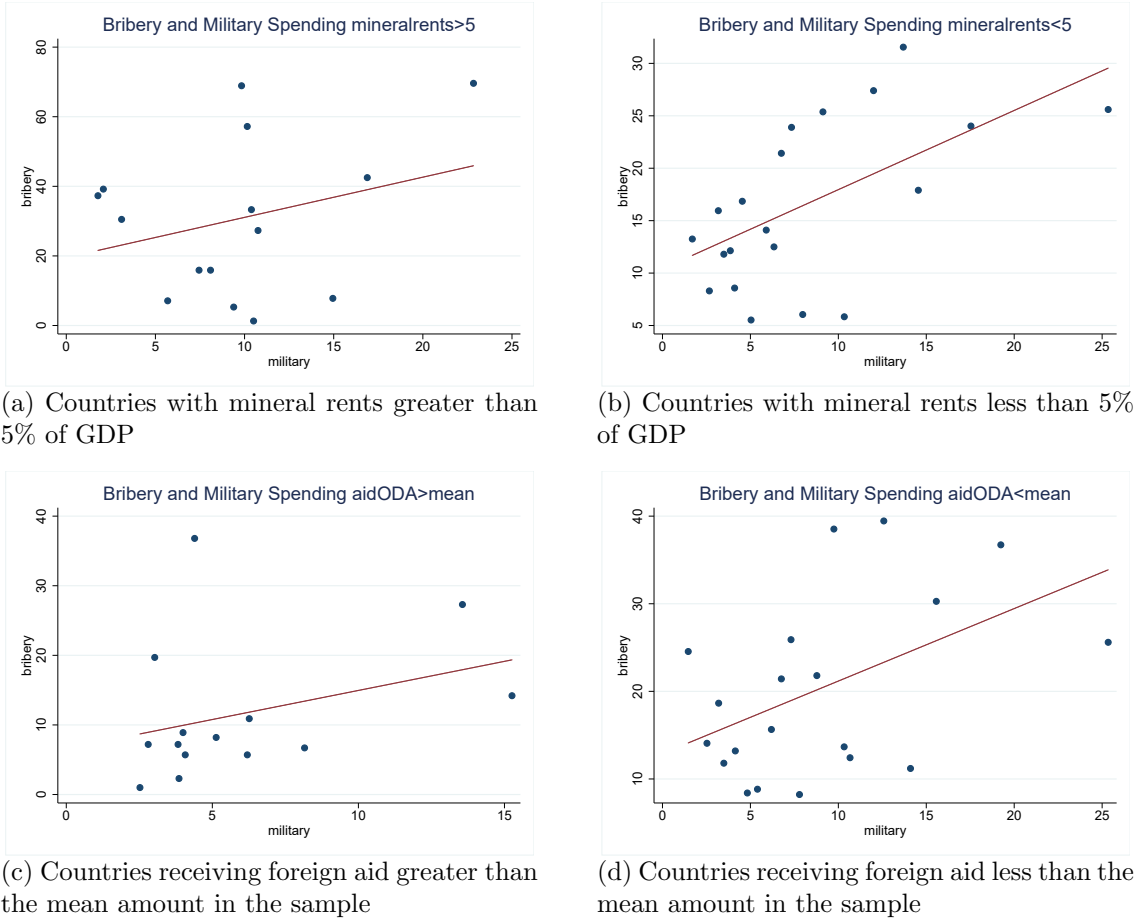
(d) Countries with oil rents less than 20% of GDP

This figure shows binned scatterplots of military spending and two alternative measures of corruption, for countries that are less reliant on oil. Data used are from a pooled cross-section of countries for which some World Development Indicators are available between years 1997 to 2012 — specifically: *bribery*, which is the percentage of firms experiencing at least one bribe payment request; *corruption*, which is the CPIA transparency, accountability, and corruption in the public sector rating (with 1 re-coded as most, and 6 least, transparent); *military*, which is military expenditure as a percentage of GDP; and *oil*, which is oil rents as a percentage of GDP

larger bribe. In this case, the bribe amounts that each candidate obtains depends on their relative ability to steal and their relative ability to increase social welfare. I show that the larger bribe goes either to the candidate who is relatively worse in both respects, or to the candidate who is relatively worse only in the ability to steal (and relatively better at increasing social welfare), provided that such disadvantage is sufficiently large. In this manner, the money (bribes) follows the candidate that is relatively worse at finding other sources of campaign funds.

To proceed with the formal analysis, let each principal $i = \{1, 2\}$ now explicitly consist of a group of individuals who choose the public official – the agent, by electing a party or candidate $k = \{A, B\}$. The game proceeds similarly, but with an additional last stage in which the agent is

Figure 5: Positive Association Between Bribery and Spending More Apparent Below the Threshold



This figure shows binned scatterplots of military spending and the incidence of bribery, for countries that are more reliant, and those that are less reliant, on mineral rents and foreign aid. Data used are from a pooled cross-section of countries for which some World Development Indicators are available between years 1997 to 2012 — specifically: *bribery*, which is the percentage of firms experiencing at least one bribe payment request; *military*, which is military expenditure as a percentage of GDP; *oil*, which is oil rents as a percentage of GDP; *mineralrents*, which is mineral rents as a percentage of GDP; and *aidODA*, which is the net official development assistance and official aid received (in constant 2012 US\$)

selected. Specifically, the leader of group 1 offers bribe/contribution schedules to each candidate, who then announce her own policy. Each member of each group then vote for either candidate *A* or *B*.

Suppose rents are now used by the candidates for campaign spending in order to sway some voters. That is, there is a fraction of total voters who are ‘impressionable’ in that they respond to such spending, while the rest are ‘strategic’ in that they vote only according to their policy preference. Let rents come from bribes and stolen revenues.

While the main model in Section 2 conducts comparative statics on the public spending and rent-seeking behavior of the incumbent agent with respect to revenues, the analyses here employ

backward induction to compare the differences in the behavior of two potential incumbents — specifically, in the level of public spending each would adopt if she were elected, and the amount of rents each would obtain. Subsection 3.1 considers the case when the amount of revenues are at or below the threshold level at which the no-theft constraint binds, in which case only bribe-rents can be used as campaign funds. Subsection 3.2 assumes that revenues are above the threshold and therefore the constraint is non-binding, in which case both bribe-rents and stolen revenues can be used to influence impressionable voters. All the analyses pertain to interior solutions.

3.1 Bribery

The setup follows Grossman and Helpman (2001). There are $N = N_1 + N_2$ voters, with $N_1 > 0$ voters belonging to group 1 and $N_2 > 0$ to group 2. Group 2 is an unorganized sector that is not capable of offering bribes, while group 1 can offer bribes. A voter can either be strategic or impressionable. A strategic voter j in group i has utility $V(g_i^k) + v_{ji}^k$, where $k = \{A, B\}$ indexes the candidate, with $V'(\cdot) > 0$, $V''(\cdot) < 0$. That is, $V(g_i^k)$ is the utility obtained from public spending to be allocated by k to group i , and is thus group-specific, whereas v_{ji}^k captures the voter's particular preference for k , and is thus voter-specific.⁸

Let $v_{ji} = v_{ji}^B - v_{ji}^A$ denote the relative preference of voter j in group i for B over candidate A . For both groups, let v_{ji} be uniformly distributed, with mean b/f and density f .

Strategic voter j in group i votes for A if and only if $v_{ji} \leq V(g_i^A) - V(g_i^B)$. This implies that the fraction of the strategic voters in group i who vote for A is:⁹

$$s_i^S = \frac{1}{2} - b + f[V(g_i^A) - V(g_i^B)] \quad (17)$$

(Thus, if both candidates adopt the same policy position, i.e. $g_i^A = g_i^B$, then $s_i^S = \frac{1}{2} - b$.) With $\sum N_i s_i^S = s^S N$, one can solve for the fraction of total strategic voters s^S who vote for A :

$$s^S = \frac{1}{2} - b + f\left[\frac{N_1}{N}[V(g_1^A) - V(g_1^B)] + \frac{N_2}{N}[V(g_2^A) - V(g_2^B)]\right]. \quad (18)$$

Now assume that for each group i , there is a fraction μ of strategic voters, and a fraction $1 - \mu$ of impressionable voters who are influenced by campaign spending. The fraction of impressionable voters in group i who vote for A when each candidate k spends bribes b^k on the campaign is:

$$s_i^I = \frac{1}{2} - b + e(b^A - b^B), \quad (19)$$

⁸In Grossman and Helpman, the strategic voter's utility is $V_i(\mathbf{g}^k) + v_{ji}^k$, where \mathbf{g} is the vector of policies, which in this case is $\mathbf{g} = (g_1, g_2)$. I simplify here by letting the first term be $V(g_i^k)$ - voter j only cares about the spending allocated to its own group, and by assuming that V takes the same functional form across groups.

⁹With mean b/f and density f , v_{ji} is uniformly distributed on the interval $\left[\frac{2b-1}{2f}, \frac{2b+1}{2f}\right]$. The share of strategic voters in group i who vote for A is thus $f[V(g_i^A) - V(g_i^B) - (\frac{2b-1}{2f})]$.

where e captures the effectiveness of such campaign spending. With s_i^I the same across groups, the vote share of A among all impressionable voters is thus

$$s^I = \frac{1}{2} - b + e(b^A - b^B). \quad (20)$$

Finally, with μ as the share of strategic, and $1 - \mu$ the share of impressionable, voters, the overall share of votes for A is the weighted share

$$s = \frac{1}{2} - b + \mu f \left[\frac{N_1}{N} [V(g_1^A) - V(g_1^B)] + \frac{N_2}{N} [V(g_2^A) - V(g_2^B)] \right] + (1 - \mu)e(b^A - b^B). \quad (21)$$

Each candidate wants to maximize its probability of winning. For A , this probability is the probability that $s > \frac{1}{2}$, which is greatest when her choice of g_i^A maximizes

$$U^A = \mu f \left[\frac{N_1}{N} V(g_1^A) + \frac{N_2}{N} V(g_2^A) \right] + (1 - \mu)eb^A. \quad (22)$$

For B , the probability that $s < \frac{1}{2}$ (or the probability that she wins) is greatest when her choice g_i^B maximizes

$$U^B = \mu f \left[\frac{N_1}{N} V(g_1^B) + \frac{N_2}{N} V(g_2^B) \right] + (1 - \mu)eb^B. \quad (23)$$

Notice that U^A and U^B are similar. More importantly, they are similar to the specification of the government's objective function U in section 2 – the weight λ that is attached to social welfare is now captured by parameters μ, f, e . Specifically, note that for the special case $\frac{N_1}{N} = \frac{N_2}{N} = \frac{1}{2}$, $U^k = \frac{\mu f}{2} [V(g_1^k) + V(g_2^k)] + (1 - \mu)eb^k$, and if $e = \frac{f}{2}$, then $U^k = \mu [V(g_1^k) + V(g_2^k)] + (1 - \mu)b^k$. In this case, the weight λ that the incumbent government attaches to social welfare in section 2 is simply motivated by the fraction μ of strategic voters.

Now recall that bribes are offered to the candidates by group 1. Group 1 thus has the problem of maximizing its members' expected benefit from g_1 , net of the bribes it gives to candidates A and/or B . However, as before, it is constrained by the requirement that A and B each attain at least their reservation utilities \overline{U}^k , i.e. when bribes are zero. That is, the bribe offer has to at least compensate each candidate from adopting a level of spending g_i that is different from the level that maximizes the welfare of the average strategic voter.

Group 1 derives total benefit $N_1 V(g_1^k)$ if k is elected, and from its view, the ex-ante probability that A is elected is $F(\Delta)$, where

$$\Delta = U^A - U^B = \mu f \left[\frac{N_1}{N} [V(g_1^A) - V(g_1^B)] + \frac{N_2}{N} [V(g_2^A) - V(g_2^B)] \right] + (1 - \mu)e(b^A - b^B). \quad (24)$$

Thus, group 1 solves:

$$\begin{aligned} & \max_{g_i^k, b^k} F(\Delta)N_1V(g_1^A) + (1 - F(\Delta))N_1V(g_1^B) - \sum_k b^k \\ & \text{s.t. } \mu f\left[\frac{N_1}{N}V(g_1^k) + \frac{N_2}{N}V(g_2^k)\right] + (1 - \mu)eb^k \geq \overline{U}^k, \end{aligned} \quad (25)$$

for each $k = \{A, B\}$. Assuming that the constraints hold with equality, one can then obtain an expression for the bribe schedule that is offered to each candidate:¹⁰

$$b^k = \left[\frac{1}{(1 - \mu)e}\right] \left[\overline{U}^k - \mu f\left[\frac{N_1}{N}V(g_1^k) + \frac{N_2}{N}V(g_2^k)\right]\right]. \quad (26)$$

Recall that when there are no theft of government revenues, $g_2 = T - g_1$. Using this fact and plugging in the expression for b^k into group 1's objective function, the group's problem can be re-cast as

$$\begin{aligned} & \max_{g_1^A, g_1^B} F(\Delta^{\mathcal{T}})N_1V(g_1^A) + (1 - F(\Delta^{\mathcal{T}}))N_1V(g_1^B) \\ & - \left[\frac{1}{(1 - \mu)e}\right] \left[\overline{U}^A + \overline{U}^B - \mu f\left[\frac{N_1}{N}(V(g_1^A) + V(g_1^B)) + \frac{N_2}{N}(V(T - g_1^A) + V(T - g_1^B))\right]\right], \end{aligned} \quad (27)$$

where $\Delta^{\mathcal{T}}$ is the same as equation (24), but now indexed by \mathcal{T} to distinguish this case as one in which theft is not possible. This yields the following FOCs:

$$V'(g_1^{A*}) = \alpha^{A*}V'(T - g_1^{A*}) \quad (28)$$

$$V'(g_1^{B*}) = \alpha^{B*}V'(T - g_1^{B*}), \quad (29)$$

where $\alpha^{A*} = \frac{\mu f \frac{N_2}{N}}{\mu f \frac{N_1}{N} - x}$, $\alpha^{B*} = \frac{\mu f \frac{N_2}{N}}{\mu f \frac{N_1}{N} + y}$, are the equilibrium 'weights' candidate A and B , respectively, attach to group 2's marginal utility from public spending, and where I have defined the following:¹¹ $x \equiv \frac{N_1 F(\Delta^{\mathcal{T}})}{N_1 \frac{\partial F}{\partial \Delta^{\mathcal{T}}} [V(g_1^{B*}) - V(g_1^{A*})] - [\frac{1}{(1 - \mu)e}]}$ and $y \equiv \frac{N_1 (1 - F(\Delta^{\mathcal{T}}))}{N_1 \frac{\partial F}{\partial \Delta^{\mathcal{T}}} [V(g_1^{B*}) - V(g_1^{A*})] + [\frac{1}{(1 - \mu)e}]}$. Also, let $\mu f \frac{N_1}{N} \neq x$, $\mu f \frac{N_1}{N} \neq -y$.

I now show how each candidate A and B would allocate total spending between groups 1 and 2 by characterizing g_1^{k*} (and, hence, $g_2^{k*} = T - g_1^{k*}$) in several ways.

First, in equilibrium, both A and B would offer not to spend all of revenues toward group 1, since each candidate attaches non-zero weight to group 2's marginal utility from public goods. That is:

¹⁰In Grossman and Helpman, the case when the constraints hold with equality is interpreted as one in which the group has a pure 'influence' motive. That is, it offers bribes in order to influence policy. They also consider the case when the constraint is a strict inequality - in this case, the group also has an 'electoral' motive in that it gives more than what is necessary to influence policy, which can then be used for greater campaign spending. They show, however, that even with electoral motives, the qualitative results are the same - bribes are offered to both candidates.

¹¹See appendix B.

Proposition 7. *Each candidate $k = \{A, B\}$ offers $g_1^{k*} < T$.*

In other words, both candidates would offer to spend on both groups, although the amount of spending would not necessarily be the same. In particular, if candidate A 's ex-ante probability of being elected is sufficiently high — higher than some threshold, then that candidate would allocate less spending to group 1 than B would (which implies that A would allocate more to group 2 than B would). The threshold decreases with the fraction $(1 - \mu)$ of impressionable voters and the size N_1 of group 1. Thus, the smaller $(1 - \mu)$ and N_1 are, the more likely it is that A would allocate less to group 1 than B would. More precisely:

Proposition 8. *Define threshold $z \equiv \frac{1}{2} - \frac{N_1 \frac{\partial F}{\partial \Delta^{\mathcal{F}}} [V(g_1^{B*}) - V(g_1^{A*})]}{(1-\mu)e}$.*

- (i) $F(\Delta^{\mathcal{F}}) > z \iff g_1^{A*} < g_1^{B*}$.
- (ii) $F(\Delta^{\mathcal{F}}) < z \iff g_1^{A*} > g_1^{B*}$.
- (iii) $F(\Delta^{\mathcal{F}}) = z \iff g_1^{A*} = g_1^{B*}$.

Proposition 8 thus implies that the relatively more popular candidate would allocate relatively less spending to group 1.¹² The intuition is that such a candidate would have less need to sway impressionable voters and, hence, less reliance on the campaign funds that group 1 offers.

I next show that the spending that is allocated to group 1 is almost always more than what is socially optimal. That is, at least one candidate would allocate to 1 an amount that exceeds that which the group would obtain if bribe offers were rejected. To do so, I first derive expressions for \bar{U}^k by letting $b^k = 0$. In this case, $U^k = \mu f[\frac{N_1}{N}V(g_1^k) + \frac{N_2}{N}V(T - g_1^k)]$ which, when maximized, yields FOCs:

$$V'(g_1^{A^0}) = \frac{N_2}{N_1}V'(T - g_1^{A^0}) \quad (30)$$

$$V'(g_1^{B^0}) = \frac{N_2}{N_1}V'(T - g_1^{B^0}). \quad (31)$$

Notice, then, that both A and B would offer the same public spending allocations if they rejected the bribes.¹³ Such allocation is socially optimal since it weighs the total marginal utilities of each

¹²To see this, note from the definition of z that z takes on values equal to, less than, or greater than, $\frac{1}{2}$ when, respectively, $V(g_1^{B*}) = V(g_1^{A*})$, $V(g_1^{B*}) > V(g_1^{A*})$, and $V(g_1^{B*}) < V(g_1^{A*})$ or, respectively, $g_1^{B*} = g_1^{A*}$, $g_1^{B*} > g_1^{A*}$, and $g_1^{B*} < g_1^{A*}$. Thus, by condition (iii), if $g_1^{B*} = g_1^{A*}$, it must be that $F(\Delta^{\mathcal{F}}) = z = \frac{1}{2} = 1 - F(\Delta^{\mathcal{F}})$ — that is, the candidates have equal probability of winning, or are equally popular. If $g_1^{B*} > g_1^{A*}$, then it must be that $z < \frac{1}{2}$. Thus, if condition (i) holds, i.e. $F(\Delta^{\mathcal{F}}) > z$, it must also be true that $F(\Delta^{\mathcal{F}}) > \frac{1}{2}$ — that is, A is more popular than B . Similarly, if $g_1^{B*} < g_1^{A*}$, then it must be that $z > \frac{1}{2}$. Thus, if condition (ii) holds, i.e. $F(\Delta^{\mathcal{F}}) < z$, it must also be true that $F(\Delta^{\mathcal{F}}) < \frac{1}{2}$ — that is, A is less popular than B .

¹³Equations (30) and (31) imply that $\frac{V'(g_1^{A^0})}{V'(T - g_1^{A^0})} = \frac{V'(g_1^{B^0})}{V'(T - g_1^{B^0})}$. Suppose that $g_1^{A^0} > g_1^{B^0}$. Then $V'(g_1^{A^0}) > V'(g_1^{B^0})$,

group only by their size, i.e. $N_1 V'(g_1^{k_0}) = N_2 V'(T - g_1^{k_0})$. Henceforth, I omit the superscript k to denote the socially optimal spending for group 1 as g_1^0 (and for group 2 as $T - g_1^0$).

Proposition 9 establishes the magnitudes of $g_1^{A^*}$ relative to the socially optimal amount g_1^0 :

Proposition 9. *Recall z and note w from Lemma 3 (below).*

- (i) $F(\Delta^{\mathcal{F}}) = z \iff g_1^{A^*} = g_1^{B^*} > g_1^0$.
- (ii) $F(\Delta^{\mathcal{F}}) < z \iff$
 - (ii.1) $g_1^{A^*} > g_1^{B^*} > g_1^0$ if $V(g_1^{A^*}) - V(g_1^{B^*}) > w$;
 - (ii.2) $g_1^{A^*} > g_1^0 > g_1^{B^*}$ if $V(g_1^{A^*}) - V(g_1^{B^*}) < w$;
- (iii) $F(\Delta^{\mathcal{F}}) > z \iff$
 - (iii.1) $g_1^{B^*} > g_1^{A^*} > g_1^0$ if $V(g_1^{B^*}) - V(g_1^{A^*}) > w$;
 - (iii.2) $g_1^{B^*} > g_1^0 > g_1^{A^*}$ if $V(g_1^{B^*}) - V(g_1^{A^*}) < w$;

Recall from Proposition 8 that if candidate A 's ex-ante probability of being elected $F(\Delta^{\mathcal{F}})$ is exactly equal to threshold z , then A and B would allocate the same amount of spending to group 1. Proposition 9 (condition (i)) establishes that this amount is greater than the socially optimal level g_1^0 , while Lemmas 2 and 3 (below) reveal that this is because both candidates attach relatively lower weight to group 1's marginal utility from spending, i.e. $\alpha^{A^*}, \alpha^{B^*} < \frac{N_2}{N_1}$, or $x < 0, y > 0$. If $F(\Delta^{\mathcal{F}}) < z$, then by Proposition 8, candidate A would spend more on group 1 than B would which, by Lemmas 2 and 3 imply that A puts higher weight to group 1's marginal utility than to 2's while B assigns higher weight to 2's marginal utility than to 1's. If $F(\Delta^{\mathcal{F}}) > z$, it is candidate B that would spend more on group 1. Nevertheless, by Proposition 9 (conditions (ii.2) and (iii.1)), both A and B would still allocate to group 1 an amount above the social optimum if the difference in the marginal value of their allocations to group 1 exceeds some threshold w .¹⁴ In turn, Lemma 3 shows that threshold w decreases with the fraction $(1 - \mu)$ of impressionable voters and the size N_1 of group 1. This implies that the larger $(1 - \mu)$ and N_1 are, the more likely it is that both A and B would overspend on this group, while the smaller these parameters, the more likely it is that only either A or B would overspend on group 1. Note, then, that it is always the case that at least one candidate would overspend on 1, which implies that, on expectation – that is, given each candidate's probability of being elected, public spending allocation would be socially inefficient.

Lemma 2. *Recall x and y .*

which requires that $V'(T - g_1^{A^0}) > V'(T - g_1^{B^0})$ and, in turn, that $(T - g_1^{A^0}) > (T - g_1^{B^0})$ or, re-arranging, $0 > g_1^{A^0} - g_1^{B^0}$. This contradicts $g_1^{A^0} > g_1^{B^0}$. One can derive an analogous contradiction for $g_1^{A^0} < g_1^{B^0}$. Thus, $g_1^{A^0} = g_1^{B^0}$, which implies that $(T - g_1^{A^0}) = (T - g_1^{B^0})$.

¹⁴Otherwise (conditions (ii.1) and (iii.2)), only A or B allocates an amount above, while the other candidate allocates below, the social optimum.

- (i) $x > 0 \iff \alpha^{A^*} > \frac{N_2}{N_1}$; (ii) $x < 0 \iff \alpha^{A^*} < \frac{N_2}{N_1}$; (iii) $x = 0 \iff \alpha^{A^*} = \frac{N_2}{N_1}$;
(iv) $y > 0 \iff \alpha^{B^*} < \frac{N_2}{N_1}$; (v) $y < 0 \iff \alpha^{B^*} > \frac{N_2}{N_1}$; (vi) $y = 0 \iff \alpha^{B^*} = \frac{N_2}{N_1}$.

Lemma 3. Define $w \equiv \frac{1}{(1-\mu)eN_1 \frac{\partial F}{\partial \Delta F}}$.

- (i) $g_1^{A^*} = g_1^{B^*} \iff x < 0, y > 0$;
(ii) $g_1^{A^*} > g_1^{B^*} \iff x < 0$, and
(ii.1) $y > 0$ if $V(g_1^{A^*}) - V(g_1^{B^*}) < w$;
(ii.2) $y < 0$ if $V(g_1^{A^*}) - V(g_1^{B^*}) > w$;
(iii) $g_1^{A^*} < g_1^{B^*} \iff y > 0$, and
(iii.1) $x > 0$ if $V(g_1^{B^*}) - V(g_1^{A^*}) > w$;
(iii.2) $x < 0$ if $V(g_1^{B^*}) - V(g_1^{A^*}) < w$.

One last characterization of $g_1^{k^*}$ can be made by using the restriction that bribes are non-negative. Plugging $\bar{U}^k = \mu f[\frac{N_1}{N}V(g_1^0) + \frac{N_2}{N}V(T - g_1^0)]$ into equation (26) gives equilibrium bribe offer by k :

$$b^{k^*} = \left[\frac{1}{(1-\mu)e} \right] \left[\mu f \left[\frac{N_1}{N} (V(g_1^{k^0}) - V(g_1^{k^*})) + \frac{N_2}{N} (V(T - g_1^{k^0}) - V(T - g_1^{k^*})) \right] \right]. \quad (32)$$

If $b^{k^*} \geq 0$, it must be that $[V(g_1^{k^*}) - V(g_1^{k^0})] \leq \frac{N_2}{N_1} [V(T - g_1^{k^0}) - V(T - g_1^{k^*})]$. Thus:

Proposition 10. The equilibrium allocation offered by candidate $k = \{A, B\}$ is such that, for each k , $\frac{V(T-g_1^0)}{V(T-g_1^{k^*})} \geq \frac{\alpha^{k^*} N_1 + N_2}{2N_2}$.

Notice that the larger group 1 is, the larger the RHS of condition $\frac{V(T-g_1^0)}{V(T-g_1^{k^*})} \geq \frac{\alpha^{k^*} N_1 + N_2}{2N_2}$ is, and the larger $g_1^{k^*}$ must be relative to g_1^0 (and, thus, the smaller $T - g_1^{k^*}$ is relative to $T - g_1^0$), in order that the LHS is sufficiently high for the condition to hold. Thus, Propositions 9 and 10 imply that social inefficiencies (from bribe-taking) are higher the larger the size of the bribing sector.

Finally, given what candidates A and B would allocate to group 1, how much bribes would the latter offer to each candidate in equilibrium?

Using (33), one can take the difference:

$$b^{A^*} - b^{B^*} = \left[\frac{1}{(1-\mu)e} \right] \mu f \left[\frac{N_1}{N} [V(g_1^{B^*}) - V(g_1^{A^*})] + \frac{N_2}{N} [V(T - g_1^{B^*}) - V(T - g_1^{A^*})] \right]. \quad (33)$$

Proposition 11 establishes that group 1 offers relatively more bribes to the candidate that would allocate less spending to the group.

Proposition 11.

- (i) $g_1^{A^*} > g_1^{B^*} \iff b^{A^*} < b^{B^*}$;

- (ii) $g_1^{A^*} < g_1^{B^*} \iff b^{A^*} > b^{B^*}$;
 (iii) $g_1^{A^*} = g_1^{B^*} \iff b^{A^*} = b^{B^*}$.

The intuition is the following. The more popular candidate has less need for campaign funds and would thus allocate less spending to the group that provides such funds, while the less popular candidate would allocate more. This would then induce the group to offer relatively larger bribes to the former and less to the latter — otherwise, the candidates would reject the bribe offers and instead choose the socially optimal public spending allocation, which is lower than the expected allocation if the bribe offers had been accepted.¹⁵ Thus, in equilibrium, the difference in bribe-offers to each candidate reflects the difference in the allocations of each candidate, given the difference in their probability of being elected, such that the expected utility of group 1 (from the expected allocations) is maximized. See Figure 6 for a graphical depiction.¹⁶

3.2 Bribery and Theft

I now consider the case when the no-theft constraint is slack, such that the candidate can also obtain rents from the theft of government revenues. To keep as close as possible to Grossman and Helpman, I let the agent use (anticipated) stolen revenues the way she uses the bribes from group 1, that is, to influence impressionable voters.¹⁷ I show that with such additional campaign funds, it is still the case that the candidate that has the higher probability of being elected would offer relatively lower spending to group 1. However, this does not have clear implications on the candidates' allocations to group 2. In the previous case in which there is no theft of revenues, a relatively lower spending allocation to group 1 leaves relatively more revenues to be spent on group 2. Now when some of the revenues can be stolen, the candidate that spends relatively less on group 1 may also steal relatively more, and thereby also spend relatively less on group 2.

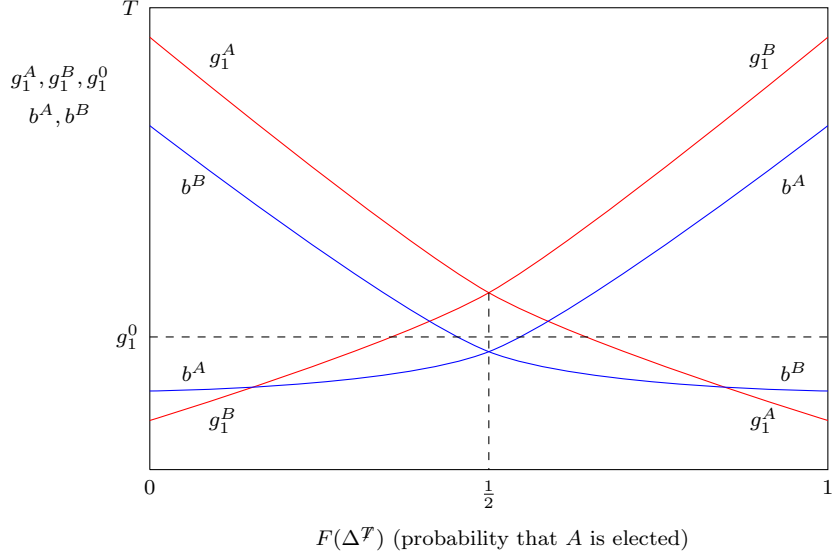
The possibility that candidates can steal different amounts also implies that, without bribes, the candidate that steals more has relatively more campaign funds. Group 1 now has to consider that candidates can have different ex-ante capabilities of influencing impressionable voters. In equilibrium, unlike in the case when the no-theft constraint is binding, the candidate that is more likely to be elected does not necessarily obtain larger bribes.

¹⁵Note, then, that cases (ii) and (iii) of Proposition 9 imply that the equilibrium probability distribution over g_1^A and g_1^B always gives an expected allocation to group 1 that is larger than g_1^0 — that is, even when one candidate allocates at a level below g_1^0 .

¹⁶The bribe curves in Figure 6 are drawn such that difference in bribe-offers to candidates A and B is a fraction of the difference in spending by A and B on group 1. It is possible, however, for the marginal utility of such spending to be sufficiently high such that the difference in bribe-offers is larger than the difference in the amounts spent on 1.

¹⁷The fact that the actual theft occurs once the agent is in office is irrelevant. Candidates either advance the 'payment' to impressionable voters, or simply promise to pay them after the election. Note that the model similarly ignores the timing of the payment of bribes, as it only solves for the equilibrium bribe *offer*.

Figure 6: Public Spending by, and Bribe-Offers to, Candidates A and B



This figure plots the amount of public spending that each candidate A and B , if elected, would allocate to group 1 (respectively depicted by red curves g_1^A and g_1^B), and the bribe-offers of this group to each candidate (depicted by blue curves b^A and b^B), on the probability $F(\Delta^T)$ that A is elected. Note that spending is always allocated to both groups 1 and 2, which is why, even at $F(\Delta^T) = 0$ and $F(\Delta^T) = 1$, g_1^A and g_1^B are between 0 and total revenues T . When the candidates have equal probability of being elected, i.e. $F(\Delta^T) = \frac{1}{2} = 1 - F(\Delta^T)$, the candidates would allocate the same amount of spending to group 1, which is higher than the socially optimal amount g_1^0 , i.e. $g_1^A = g_1^B > g_1^0$. When candidate A has a relatively lower probability of being elected, she would allocate to group 1 an amount that is higher than what B would allocate, and higher than what is socially optimal. (Thus, in the region where $F(\Delta^T) < \frac{1}{2}$, either $g_1^A > g_1^B > g_1^0$ or $g_1^A > g_1^0 > g_1^B$.) This would enable group 1 to offer less bribes to A , and more to B , i.e. $b^A < b^B$, thereby maximizing its expected utility from the spending allocations of each candidate. (An analogous pattern holds in the region where $F(\Delta^T) > \frac{1}{2}$.)

To proceed with the formal analysis, I now add stolen revenues $T - g_1^k - g_2^k$ to each candidate k 's campaign funds such that the vote share of A among impressionable voters is $s^I = \frac{1}{2} - b + e(b^A - b^B + (T - g_1^A - g_2^A) - (T - g_1^B - g_2^B)) = \frac{1}{2} - b + e(b^A - b^B + g_1^B - g_1^A + g_2^B - g_2^A)$. The total vote share of A is now:

$$s = \frac{1}{2} - b + \mu f \left[\frac{N_1}{N} [V(g_1^A) - V(g_1^B)] + \frac{N_2}{N} [V(g_2^A) - V(g_2^B)] \right] + (1 - \mu) e (b^A - b^B + g_1^B - g_1^A + g_2^B - g_2^A). \quad (34)$$

Thus, A and B 's respective probability of winning are highest when the following are maximized:

$$U^A = \mu f \left[\frac{N_1}{N} V(g_1^A) + \frac{N_2}{N} V(g_2^A) \right] + (1 - \mu) e (b^A - g_1^A - g_2^A) \quad (35)$$

$$U^B = \mu f \left[\frac{N_1}{N} V(g_1^B) + \frac{N_2}{N} V(g_2^B) \right] + (1 - \mu) e (b^B - g_1^B - g_2^B), \quad (36)$$

and the ex-ante probability that A is elected is $F(\Delta)$ where, now,

$$\Delta = U^A - U^B = \mu f \left[\frac{N_1}{N} [V(g_1^A) - V(g_1^B)] + \frac{N_2}{N} [V(g_2^A) - V(g_2^B)] \right] + (1 - \mu) e (b^A - b^B - (g_1^A + g_2^A) + (g_1^B + g_2^B)). \quad (37)$$

Group 1 thus solves:

$$\begin{aligned} & \max_{g_i^k, b^k} F(\Delta) N_1 V(g_1^A) + (1 - F(\Delta)) N_1 V(g_1^B) - \sum_k b_T^k \\ \text{s.t. } & \mu f \left[\frac{N_1}{N} V(g_1^k) + \frac{N_2}{N} V(g_2^k) \right] + (1 - \mu) e (b_T^k - g_1^k - g_2^k) \geq \overline{U}_T^k \quad (a) \\ & g_1^k + g_2^k \leq T \quad (b), \end{aligned} \quad (38)$$

for each $k = \{A, B\}$, and bribes b_T^k and the agent's reservation utility \overline{U}_T^k are subscripted by T to distinguish the case when theft can occur.

Note that when constraint (b) is binding, Δ collapses back to equation (24), and the optimization problem reduces to (27) - the case of no theft, by letting $g_1^k = T - g_2^k$.

To see this, note that one gets the following expression for bribes by letting constraint (a) bind with equality:

$$b_T^k = \left[\frac{1}{(1 - \mu)e} \right] \left[\overline{U}_T^k - \mu f \left[\frac{N_1}{N} V(g_1^k) + \frac{N_2}{N} V(g_2^k) \right] \right] + g_1^k + g_2^k, \quad (39)$$

where the reservation utilities are given by $\overline{U}_T^k = \mu f \left[\frac{N_1}{N} V(g_1^{k^0}) + \frac{N_2}{N} V(g_2^{k^0}) \right] - (1 - \mu) e (g_1^{k^0} + g_2^{k^0})$, i.e. when bribes are rejected. Plugging the expression in (39) into the maximand of (38), the problem then becomes:

$$\begin{aligned} & \max_{g_1^k, g_2^k} F(\Delta) N_1 V(g_1^A) + (1 - F(\Delta)) N_1 V(g_1^B) \\ & - \left[\frac{1}{(1 - \mu)e} \right] \left[(\overline{U}_T^A + \overline{U}_T^B) - \mu f \left[\frac{N_1}{N} (V(g_1^A) + V(g_1^B)) + \frac{N_2}{N} (V(g_2^A) + V(g_2^B)) \right] \right] - (g_1^A + g_2^A) - (g_1^B + g_2^B), \\ \text{s.t. } & g_1^A + g_2^A - T \leq 0; \quad g_1^B + g_2^B - T \leq 0. \end{aligned} \quad (40)$$

The previous no-theft case is the special instance when the constraints bind, i.e. $g_1^k = T - g_2^k$, \overline{U}_T^k becomes $\overline{U}^k = \mu f \left[\frac{N_1}{N} V(g_1^{k^0}) + \frac{N_2}{N} V(T - g_1^{k^0}) \right]$, and $F(\Delta)$ becomes $F(\Delta^{\mathcal{T}})$, in which case solving (40) is equivalent to solving (27).¹⁸

¹⁸That is, the problem becomes $\max_{g_1^A, g_1^B} F(\Delta^{\mathcal{T}}) N_1 V(g_1^A) + (1 - F(\Delta^{\mathcal{T}})) N_1 V(g_1^B) - \left[\frac{1}{(1 - \mu)e} \right] \left[(\overline{U}^A + \overline{U}^B) - \mu f \left[\frac{N_1}{N} (V(g_1^A) + V(g_1^B)) + \frac{N_2}{N} (V(T - g_1^A) + V(T - g_1^B)) \right] \right] - 2T$, whose solution is the same as that of (27).

To obtain the equilibrium when the constraints are slack and, thus, theft occurs, I derive the Kuhn-Tucker conditions from (40):¹⁹

$$N_1 F(\Delta) V'(g_1^{A*}) + 1 - \lambda^{A*} - \frac{\partial \Delta}{\partial g_1^{A*}} \left[N_1 \frac{\partial F}{\partial \Delta} [V(g_1^{B*}) - V(g_1^{A*})] - \frac{1}{(1-\mu)e} \right] = 0 \quad (41)$$

$$1 - \lambda^{A*} - \frac{\partial \Delta}{\partial g_2^{A*}} \left[N_1 \frac{\partial F}{\partial \Delta} [V(g_1^{B*}) - V(g_1^{A*})] - \frac{1}{(1-\mu)e} \right] = 0 \quad (42)$$

$$N_1 (1 - F(\Delta)) V'(g_1^{B*}) - 1 - \lambda^{B*} - \frac{\partial \Delta}{\partial g_1^{B*}} \left[N_1 \frac{\partial F}{\partial \Delta} [V(g_1^{B*}) - V(g_1^{A*})] + \frac{1}{(1-\mu)e} \right] = 0 \quad (43)$$

$$-1 - \lambda^{B*} - \frac{\partial \Delta}{\partial g_2^{B*}} \left[N_1 \frac{\partial F}{\partial \Delta} [V(g_1^{B*}) - V(g_1^{A*})] + \frac{1}{(1-\mu)e} \right] = 0 \quad (44)$$

$$\lambda^{A*} (g_1^{A*} + g_2^{A*} - T) = 0 \quad (45)$$

$$\lambda^{B*} (g_1^{B*} + g_2^{B*} - T) = 0, \quad (46)$$

where λ^{k*} are the Lagrange multipliers.

Imposing $\lambda^{A*} = 0$, (45) implies that $g_1^{A*} + g_2^{A*} < T$ while (41) and (42) imply that (i) $N_1 = \left(\frac{\partial g_2^{A*}}{\partial g_1^{A*}} - 1 \right) \left(\frac{1}{V'(g_1^{A*}) F(\Delta)} \right)$, where $\frac{\partial g_2^{A*}}{\partial g_1^{A*}} = \frac{\partial \Delta}{\partial g_1^{A*}} \frac{\partial g_2^{A*}}{\partial \Delta} = \frac{\mu f \frac{N_1}{N} V'(g_1^{A*}) - (1-\mu)e}{\mu f \frac{N_2}{N} V'(g_2^{A*}) - (1-\mu)e}$. Imposing $\lambda^{B*} = 0$, (46) implies that $g_1^{B*} + g_2^{B*} < T$ while (43) and (44) imply that (ii) $N_1 = \left(1 - \frac{\partial g_2^{B*}}{\partial g_1^{B*}} \right) \left(\frac{1}{V'(g_1^{B*}) (1-F(\Delta))} \right)$, where $\frac{\partial g_2^{B*}}{\partial g_1^{B*}} = \frac{\partial \Delta}{\partial g_1^{B*}} \frac{\partial g_2^{B*}}{\partial \Delta} = \frac{-\mu f \frac{N_1}{N} V'(g_1^{B*}) + (1-\mu)e}{-\mu f \frac{N_2}{N} V'(g_2^{B*}) + (1-\mu)e}$. Equating (i) and (ii) and re-arranging, the equilibrium when theft occurs thus satisfies:

$$\frac{V'(g_1^{A*})}{V'(g_1^{B*})} = \frac{(1 - F(\Delta)) \left(\frac{\partial g_2^{A*}}{\partial g_1^{A*}} - 1 \right)}{F(\Delta) \left(1 - \frac{\partial g_2^{B*}}{\partial g_1^{B*}} \right)} \quad (47)$$

The following results are readily obtained.

Proposition 12. *Both candidates offer to allocate some spending on each sector. That is, $g_i^{k*} > 0$ for $i = \{1, 2\}, k = \{A, B\}$.*

Lemma 4. *For each $k = \{A, B\}$, $\frac{\partial g_2^{k*}}{\partial g_1^{k*}} \neq |1|$. If $\frac{\partial g_2^{A*}}{\partial g_1^{A*}} \geq 1$, then $\frac{\partial g_2^{B*}}{\partial g_1^{B*}} \leq 1$, and vice-versa.*

Proposition 13. *Let $w \equiv \left(\frac{\partial g_2^{A*}}{\partial g_1^{A*}} - 1 \right) \left(\frac{\partial g_2^{A*}}{\partial g_1^{A*}} - \frac{\partial g_2^{B*}}{\partial g_1^{B*}} \right)$.*

(i) $F(\Delta) > w \iff g_1^{A} < g_1^{B*}$.*

(ii) $F(\Delta) < w \iff g_1^{A} > g_1^{B*}$.*

(iii) $F(\Delta) = w \iff g_1^{A} = g_1^{B*}$.*

¹⁹See appendix C for the derivation of (41) to (44).

Proposition 13 implies that the candidate that has a relatively higher probability of being elected would allocate relatively more spending to group 1.²⁰ This result is similar to the case when the no-theft constraint is binding – recall Proposition 8. Now, however, both candidates would either overspend on group 1, or on group 2, depending on the relative size of the groups, as established by Proposition 14 and Corollary 2 below. In the previous case when the only source of campaign funds are bribe-rents and, hence, group 1, at least one candidate overspends on group 1. Now that rents from theft can also be used to buy votes, there is less dependence on group 1, which allows both candidates to cater more to group 2 as the latter becomes large.

Proposition 14. *Neither candidate offers the socially optimal allocation. Both of them either overspend on group 1 or group 2.*

Note that when N_1 is large, it is easier to meet condition $\frac{V'(g_1^k)}{V'(g_2^k)} > \frac{V'(g_1^0)}{V'(T-g_1^0)} = \frac{N_2}{N_1}$ than $\frac{V'(g_1^k)}{V'(g_2^k)} < \frac{V'(g_1^0)}{V'(T-g_1^0)} = \frac{N_2}{N_1}$. Thus:

Corollary 2. *Both candidates are more likely to overspend on group 1 than on group 2 the larger the size of the former.*

To complete the analysis, I now compare the equilibrium bribe offers to candidates A and B. Plugging \overline{U}_T^k into (39) to get

$$b_T^{k*} = \left[\frac{1}{(1-\mu)e} \right] \left[\mu f \left[\frac{N_1}{N} (V(g_1^{k_0}) - V(g_1^{k*})) + \frac{N_2}{N} (V(g_2^{k_0}) - V(g_2^{k*})) \right] + g_1^{k*} + g_2^{k*} - (g_1^{k_0} + g_2^{k_0}) \right], \quad (48)$$

one can take the difference:

$$\begin{aligned} b_T^{A*} - b_T^{B*} = & \left[\frac{1}{(1-\mu)e} \right] \left[\mu f \left[\frac{N_1}{N} (V(g_1^{A_0}) - V(g_1^{A*}) + V(g_1^{B*}) - V(g_1^{B_0})) \right. \right. \\ & \left. \left. + \frac{N_2}{N} (V(g_2^{A_0}) - V(g_2^{A*}) + V(g_2^{B*}) - V(g_2^{B_0})) \right] \right] \\ & + (g_1^{A*} + g_2^{A*}) - (g_1^{A_0} + g_2^{A_0}) + (g_1^{B_0} + g_2^{B_0}) - (g_1^{B*} + g_2^{B*}). \end{aligned} \quad (49)$$

It is not always the case that $g_1^{A_0} = g_1^{B_0}$ and $g_2^{A_0} = g_2^{B_0}$, since $g_i^{k_0}$ only requires $\frac{V'(g_1^{k_0})}{V'(g_2^{k_0})} = \frac{N_2}{N_1}$ for each $k = \{A, B\}$.²¹ However, the latter implies that if $g_1^{A_0} = g_1^{B_0}$, then $g_2^{A_0} = g_2^{B_0}$, and vice versa.

²⁰To see this, note that $F(\Delta) \geq w \iff 1 - F(\Delta) \leq 1 - w$, while $F(\Delta) = w \iff 1 - F(\Delta) = 1 - w$. Now it must be that w is between 0 and 1. (Otherwise, if $w < 0$ or $w > 1$, then $1 - F(\Delta) > 1 - w$ and $F(\Delta) < w$ cannot both be true.) This implies that when $g_1^{A*} = g_1^{B*}$, for both $F(\Delta) = w$ and $1 - F(\Delta) = 1 - w$ to be true, it must be that $w = \frac{1}{2}$, which means that $F(\Delta) = \frac{1}{2} = 1 - F(\Delta)$. That is, candidates A and B have equal probability of being elected. Now, when $g_1^{A*} > g_1^{B*}$, for both $F(\Delta) < w$ and $1 - F(\Delta) > 1 - w$ to be true when $w \in (0, 1)$, it must be that $F(\Delta) < 1 - F(\Delta)$. Analogously, when $g_1^{A*} < g_1^{B*}$, for both $F(\Delta) > w$ and $1 - F(\Delta) < 1 - w$ to be true when $w \in (0, 1)$, it must be that $F(\Delta) > 1 - F(\Delta)$.

²¹If k were to reject the bribe, she would choose $g_i^{k_0}$ by solving $\max_{g_1^k, g_2^k} U_T^k = \mu f \left[\frac{N_1}{N} V(g_1^{k_0}) + \frac{N_2}{N} V(g_2^{k_0}) \right] - (1 - \mu)e(g_1^{k_0} + g_2^{k_0})$, s.t. $g_1^{k_0} + g_2^{k_0} \leq T$ when the constraint is slack, which yields $\frac{V'(g_1^{k_0})}{V'(g_2^{k_0})} = \frac{N_2}{N_1}$.

The following result thus only needs to assume that there would be no difference in the candidates' behavior toward principal 1 if they were to reject the latter's offer, i.e. $g_1^{A^0} = g_1^{B^0}$.

Proposition 15. *Let $g_1^{A^0} = g_1^{B^0}$. Define $x \equiv (T - g_1^{A^*} - g_2^{A^*}) - (T - g_1^{B^*} - g_2^{B^*}) = (g_1^{B^*} + g_2^{B^*}) - (g_1^{A^*} + g_2^{A^*})$ as the difference in stolen revenues from electing candidate A over B, and $y \equiv [\frac{N_1}{N}V(g_1^{B^*}) + \frac{N_2}{N}V(g_2^{B^*})] - [\frac{N_1}{N}V(g_1^{A^*}) + \frac{N_2}{N}V(g_2^{A^*})]$ as the difference in social welfare from electing candidate B over A. Then:*

- (i) $x < [\frac{\mu f}{(1-\mu)e}]y \iff b_T^{A^*} > b_T^{B^*}$.
- (ii) $x > [\frac{\mu f}{(1-\mu)e}]y \iff b_T^{A^*} < b_T^{B^*}$.
- (iii) $x = [\frac{\mu f}{(1-\mu)e}]y \iff b_T^{A^*} = b_T^{B^*}$.

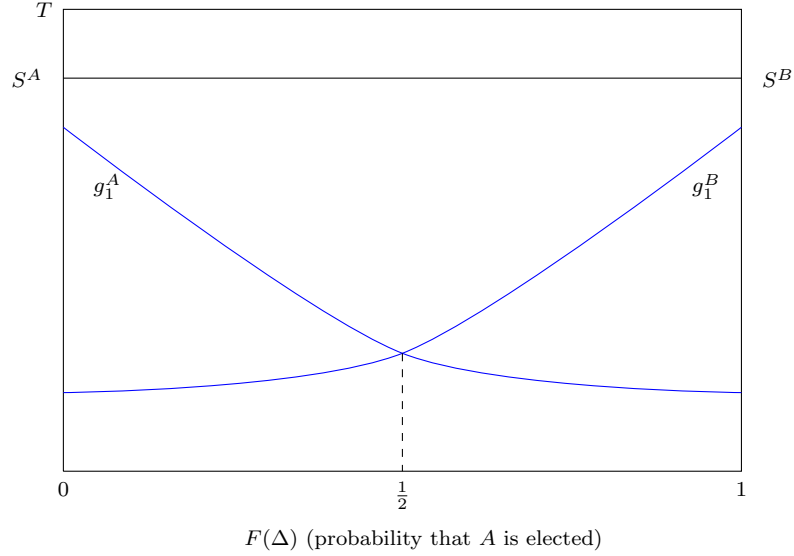
Thus, by Proposition 15, bribes augment stolen revenues such that the candidate that obtains larger bribes is either: (i) one who is relatively worse both in her ability to steal revenues and the ability to increase social welfare, i.e. $x < 0$ and $y > 0$, or, $x > 0$ and $y < 0$; or, when one is relatively worse in one respect but better in the other, i.e. $x, y < 0$ or $x, y > 0$, (ii) to the candidate who is less able to steal revenues provided that the relative disadvantage is sufficiently high, i.e. $\frac{x}{y} > \frac{\mu f}{(1-\mu)e} \equiv \bar{\mu}$. To see the latter, note that if $x, y < 0$, A is worse at stealing but better at improving social welfare. In this case, Proposition 15 implies that $\frac{x}{y} > \bar{\mu} \iff b_T^A > b_T^B$ — A obtains higher bribes. If $x, y > 0$, B is worse at stealing but better at improving social welfare. In this case, $\frac{x}{y} > \bar{\mu} \iff b_T^A < b_T^B$ — B obtains higher bribes.

In what follows, I graphically depict the results established by Propositions 13 and 15. First, Figures 7 and 8 illustrate the equilibrium public spending allocations of candidates A and B. By Proposition 13, the candidate with the higher probability of being elected allocates relatively less spending to group 1. Note, then, that the g_1^A curve lies below (above) the g_1^B curve at values of $F(\Delta)$ greater (less) than $\frac{1}{2}$, with the curves intersecting at $F(\Delta) = \frac{1}{2}$. Since theft is now possible, the total amount of spending $S^k \equiv g_1^k + g_2^k$ of each candidate $k \in \{A, B\}$ need not equal revenues T — the S^A and S^B curves can lie below T . Thus, unlike in Figure 6 — the no-theft case where the amount allocated to group 2 is the distance between T and the g_1^k curve, here it is the distance between the S^k and g_1^k curves. This means that $g_1^A < g_1^B$ does not imply $g_2^A > g_2^B$, precisely because the candidates may differ in the total amounts S^k that each would spend and, therefore, in the amounts $T - S^k$ that each would steal.

I depict two special cases. In Figure 7, the candidates always steal the same amount of revenues, in which case the spending curves S^A and S^B intersect at all values of $F(\Delta)$. Notice that a relatively higher allocation to group 1 implies a relatively lower allocation to group 2. Thus, as in the case when the no-theft constraint is binding, $F(\Delta) \leq \frac{1}{2} \iff g_1^A \geq g_1^B \iff g_2^A \leq g_2^B$. In Figure 8, the candidates steal an amount that is each a fixed proportion of each of their allocations to group 1

or, equivalently, $S^k \propto g_1^k$. In this case, S^A and S^B intersect at a unique point – panel (a) shows them intersecting at the point at which $F(\Delta) = A_1$, with $A_1 \in [0, \frac{1}{2})$, while panel (b) shows the intersection at $F(\Delta) = A_2$, with $A_2 \in (\frac{1}{2}, 1]$.²² Notice that when the spending curves intersect at $F(\Delta) = A_1$, then $g_1^A < g_1^B$ at all values of $F(\Delta)$. When they intersect at $F(\Delta) = A_2$, then $g_2^A > g_2^B$ at all values of $F(\Delta)$. That is, the candidate that steals at a higher rate, i.e. for which the vertical distance between S^k and g_1^k is smaller, always allocates relatively less spending to group 2.

Figure 7: Public Spending by Candidates A and B , with A and B Stealing the Same Amount of Revenues

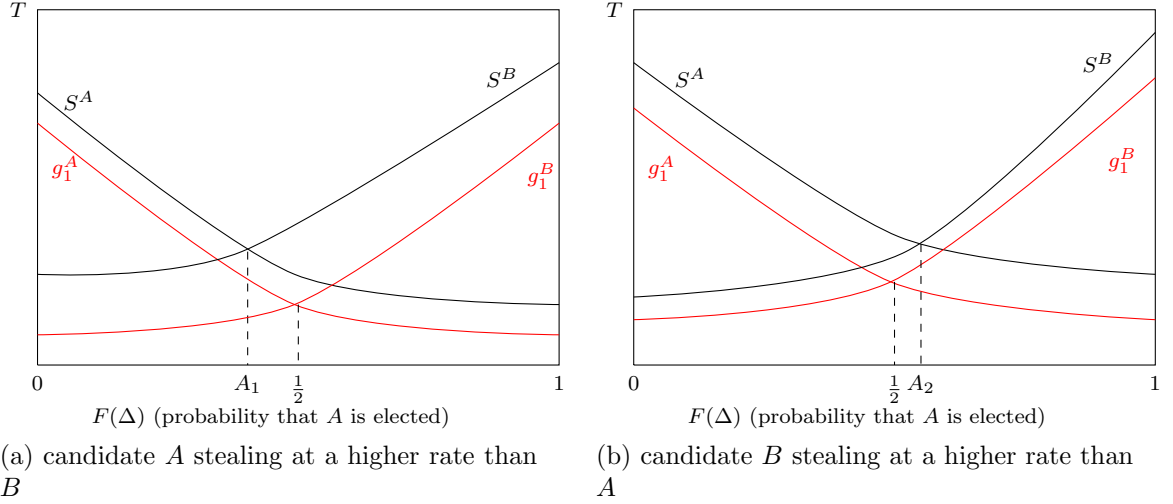


This figure plots the amount of public spending that each candidate A and B , if elected, would allocate to group 1 (respectively depicted by red curves g_1^A and g_1^B) on the probability $F(\Delta)$ that A is elected, assuming that A and B would always spend the same total amount, i.e. the total spending curves $S^A \equiv g_1^A + g_2^A$ and $S^B \equiv g_1^B + g_2^B$ intersect at all values of $F(\Delta)$. (The amounts A and B would each allocate to group 2, i.e. g_2^A and g_2^B , are given by the vertical distance between S^A and g_1^A , and between S^B and g_1^B , respectively.) This means that the candidates would also steal the same amount of revenues, given by the vertical distance between revenues T and S^A or S^B . The candidate that has relatively lower probability of being elected would allocate relatively more spending to group 1 – when $F(\Delta)$ is less (greater) than $\frac{1}{2}$, the g_1^A curve lies above (below) the g_1^B curve. At $F(\Delta) = \frac{1}{2}$, $g_1^A = g_1^B$. Because total spending is the same for both candidates, the reverse pattern holds for g_2^A, g_2^B , i.e. $g_2^A \geq g_2^B$ when $F(\Delta) \leq \frac{1}{2}$.

Next, I infer the equilibrium amount of bribes that group 1 would offer to candidates A and B using Proposition 15. In Figure 7, $S^A = S^B$ and, at $F(\Delta) = \frac{1}{2}$, $g_1^A = g_1^B$. Thus, x and y from Proposition 15 are equal to zero, which implies that group 1 offers the same amount of bribes to the candidates, i.e. $b_T^A = b_T^B$. In the region $F(\Delta) \in (\frac{1}{2}, 1]$, $g_1^A < g_1^B$. Since $S^A = S^B$, then $x = 0$

²²There are many other equilibria depending on how much revenues each candidate would steal at each value of $F(\Delta)$. The S^k curves may not intersect, or intersect at multiple points.

Figure 8: Public Spending by Candidates A and B , with A and B Stealing at Fixed Rates



This figure plots the amount of public spending that each candidate A and B , if elected, would allocate to group 1 (respectively depicted by red curves g_1^A and g_1^B) on the probability $F(\Delta)$ that A is elected, assuming that each steals an amount that is each a fixed proportion of each of their allocations to group 1. In panel (a), A steals at a higher rate than B , such that the candidates' respective total spending curves $S^A \equiv g_1^A + g_2^A$ and $S^B \equiv g_1^B + g_2^B$ intersect at $F(\Delta) = A_1 < \frac{1}{2}$, while in panel (b), where B steals at a higher rate than A , the spending curves intersect at $A_2 > \frac{1}{2}$. The amounts that each candidate would allocate to group 2, i.e. g_2^A and g_2^B , are given by the respective distances between S^A and g_1^A , and between S^B and g_1^B . Notice, then, that the candidate that steals at a higher rate spends relatively less on group 2. From (a), when S^A and S^B intersect (only) at a value of $F(\Delta)$ that is less than $\frac{1}{2}$, it is always the case that $g_2^A < g_2^B$. From (b), when the point of intersection is at some value of $F(\Delta)$ greater than $\frac{1}{2}$, then $g_2^A > g_2^B$. As for the spending on group 1, it is still the case (as in Figure 7) that when $F(\Delta) \geq \frac{1}{2}$, $g_1^A \leq g_1^B$.

and, in addition, because g_1^A and g_1^B are symmetric, then $|g_1^A - g_1^B| = |g_2^A - g_2^B|$, which means $y = 0$. Thus, with $x, y = 0$, $b_T^A = b_T^B$. Lastly, when $F(\Delta) \in [0, \frac{1}{2})$, $g_1^A > g_1^B$, but since $S^A = S^B$ and g_1^A and g_1^B are symmetric, it is still the case that both x and y are zero and, hence, $b_T^A = b_T^B$. Thus, if the candidates would always steal the same amount, they would always obtain the same amount of bribes. This is because if the bribe offers are rejected, the candidates would still get the same amount of rents (in the form of stolen revenues) and, thus, still have the same ability to sway impressionable voters.²³

In Figure 8, where each candidate steals at a fixed rate, S^A is equal to S^B only at some unique value of $F(\Delta)$ – that is, at $A_1 < \frac{1}{2}$ in panel (a), and $A_2 > \frac{1}{2}$ in panel (b). Recall that when S^A and S^B intersect at a value of $F(\Delta)$ less (greater) than $\frac{1}{2}$, then it is always the case that $g_2^A < g_2^B$ ($g_2^A > g_2^B$). Now in panel (a), note that at $F(\Delta) \in [0, A_1)$, $S^A > S^B$, which means that $x < 0$. It is also the case that $g_1^A > g_1^B$. For $x < 0$ to hold, it must be that $g_1^B - g_1^A < g_2^A - g_2^B$ which,

²³Note, then, from equation (49) that when $S^A = S^B$ (and recalling the assumption $g_1^{A^0} = g_1^{B^0}$ in Proposition 15), the difference $b_T^A - b_T^B$ is equal to zero.

with $g_2^A < g_2^B$, implies that $y > 0$. Since $x < 0$ and $y > 0$, then $b_T^A > b_T^B$ by Proposition 15. At $F(\Delta) = A_1$, $S^A = S^B$, which means $x = 0$. It is still the case that $g_1^A > g_1^B$. For $x = 0$ to hold, it must be that $g_1^B - g_1^A = g_2^A - g_2^B$ which, with $g_2^A < g_2^B$, implies that $y = 0$. Thus, $b_T^A = b_T^B$. At $F(\Delta) \in (A_1, \frac{1}{2})$, $S^A < S^B$, which means $x > 0$. It is still the case that $g_1^A > g_1^B$. For $x > 0$ to hold, it must be that $g_1^B - g_1^A > g_2^A - g_2^B$ which, with $g_2^A < g_2^B$, implies that $y < 0$. Thus, $b_T^A < b_T^B$. Finally, at $F(\Delta) \in (\frac{1}{2}, 1]$, $S^A < S^B$ and, hence, $x > 0$, which in turn requires $g_1^B - g_1^A > g_2^A - g_2^B$. However, it is now the case that $g_1^A < g_1^B$. Thus, for $g_1^B - g_1^A > g_2^A - g_2^B$ to hold when $g_2^A < g_2^B$, it must be that $y > 0$. With $x, y > 0$, $b_T^A \leq b_T^B$ if $\frac{x}{y} \geq \frac{\mu f}{(1-\mu)e} \equiv \bar{\mu}$, while $b_T^A = b_T^B$ if $\frac{x}{y} = \bar{\mu}$, where $\bar{\mu}$ is some threshold ratio of one candidate's relative ability to steal to the other candidate's relative ability to improve social welfare.

By symmetry, the case when S^A and S^B intersect at A_2 — see panel (b), can be viewed from B 's perspective as the case when S^A and S^B intersect at A_1 . Thus, the following summarizes the results for the special case in which candidates steal an amount of revenues that is each a fixed proportion of each candidate's allocations to group 1.

Let $S^A = S^B$ (only) at $F(\Delta) = A_1$, where $A_1 \in [0, \frac{1}{2})$. That is, candidate A steals at a higher rate than B . Then:

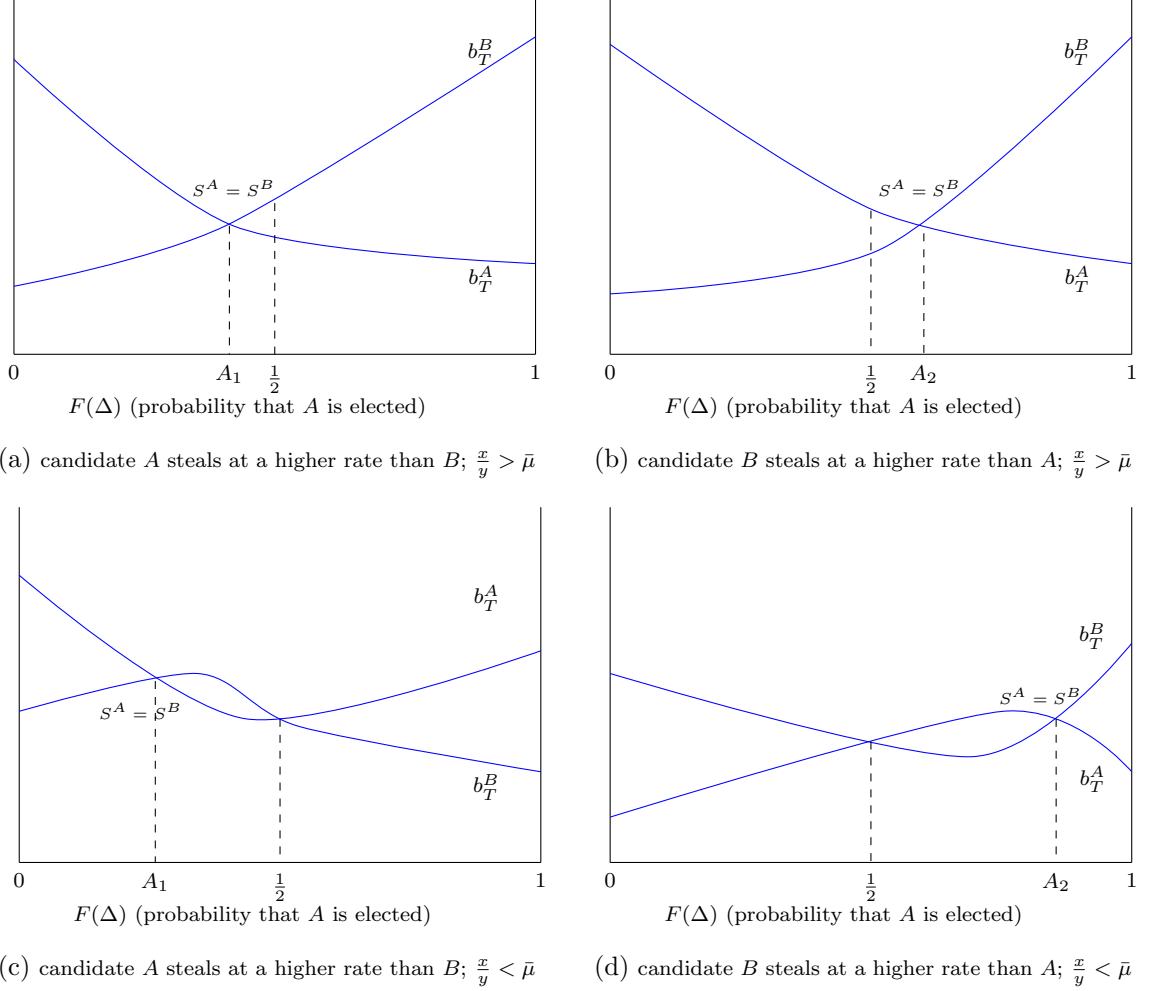
$$\begin{aligned}
0 &\leq F(\Delta) < A_1 \iff g_1^A > g_1^B, S^A > S^B \iff b_T^A > b_T^B \\
&F(\Delta) = A_1 \iff g_1^A > g_1^B, S^A = S^B \iff b_T^A = b_T^B \\
A_1 &< F(\Delta) \leq \frac{1}{2} \iff g_1^A \geq g_1^B, S^A < S^B \iff b_T^A < b_T^B \\
\frac{1}{2} &< F(\Delta) \leq 1 \iff g_1^A < g_1^B, S^A < S^B \iff b_T^A \leq b_T^B \text{ if } \frac{x}{y} \geq \bar{\mu} \text{ (otherwise } b_T^A = b_T^B)
\end{aligned}$$

Let $S^A = S^B$ (only) at $F(\Delta) = A_2$, where $A_2 \in (\frac{1}{2}, 1]$. Then, by symmetry:

$$\begin{aligned}
0 &\leq F(\Delta) < \frac{1}{2} \iff g_1^A > g_1^B, S^A > S^B \iff b_T^A \geq b_T^B \text{ if } \frac{x}{y} \geq \bar{\mu} \text{ (otherwise } b_T^A = b_T^B) \\
\frac{1}{2} &\leq F(\Delta) < A_2 \iff g_1^A \leq g_1^B, S^A > S^B \iff b_T^A > b_T^B \\
&F(\Delta) = A_2 \iff g_1^A < g_1^B, S^A = S^B \iff b_T^A = b_T^B \\
A_2 &< F(\Delta) \leq 1 \iff g_1^A < g_1^B, S^A < S^B \iff b_T^A < b_T^B
\end{aligned}$$

Figure 9 illustrates these results by plotting the relationship between the probability $F(\Delta)$ of A being elected and the bribes offered to A and B in cases in which S^A and S^B intersect (uniquely)

Figure 9: Bribe-Offers to Candidates A and B



This figure plots the bribe-offers of group 1 to each candidate A and B (respectively depicted by blue curves b_T^A and b_T^B) on the probability $F(\Delta)$ that A is elected, assuming that A and B steal an amount of revenues that is each a fixed proportion of each candidate's allocation to group 1. That is, there is a unique value of $F(\Delta)$ at which A and B would spend the same amount, i.e. $S^A = S^B$. Below (above) this value, S^A is greater (less) than S^B and, hence, A would steal less (more) than B (in absolute amounts). Panels (a) and (b) illustrate the case when the ratio of A 's relative ability to steal revenues, x , to B 's relative ability to improve social welfare, y , is larger than threshold $\bar{\mu}$, i.e. $\frac{x}{y} > \bar{\mu}$, with (a) depicting the case when S^A and S^B intersect at $F(\Delta) = A_1 < \frac{1}{2}$, and (b) at $F(\Delta) = A_2 > \frac{1}{2}$. Panels (c) and (d) are when $\frac{x}{y} < \bar{\mu}$, with (c) depicting the case when S^A and S^B intersect at $F(\Delta) = A_1 < \frac{1}{2}$, and (d) at $F(\Delta) = A_2 > \frac{1}{2}$.

at any value of $F(\Delta)$, i.e. at A_1 , or A_2 , where $A_1 \in [0, \frac{1}{2})$ and $A_2 \in (\frac{1}{2}, 1]$, and when the ratio of a candidate's relative ability to steal to its relative ability to improve social welfare, i.e. $\frac{x}{y}$, is higher, and when it is lower, than threshold $\bar{\mu}$.

Notice, then, that the candidate that is more likely to win does not always obtain larger bribes. In panels (a) and (c), candidate A receives more bribes than B when $F(\Delta)$ is between 0 and A_1 , that is, when A has lower probability of being elected than B . Analogously, as seen in panels (b) and (d), candidate B obtains larger bribes even if its probability of winning is less than $\frac{1}{2}$, i.e. when $F(\Delta)$ is between A_2 and 1.

The results are a stark contrast to the case in which the no-theft constraint binds. When the candidates' only source of campaign funds are bribes, the candidate that is more likely to be elected always obtains more bribes. However, this is not necessarily true when candidates can also use stolen revenues to sway impressionable voters. Generally, larger bribes are given to the candidate that is either disadvantaged both in the relative ability to steal and the relative ability to improve social welfare, i.e. $x < 0, y > 0$ or $x > 0, y < 0$ or, if one candidate is relatively worse in one respect but better in the other, to the candidate that is relatively worse at stealing, provided that such relative disadvantage is sufficiently high, i.e. $\frac{x}{y} > \bar{\mu}$.

4 Conclusion

This paper formally analyzes public-good spending by a politician who can obtain rents by stealing government revenues or spending those revenues in exchange for bribes. To the best of my knowledge, the model I have proposed is the first to simultaneously consider these two types of corruption. The analysis generates several important results.

The relationship between government revenues, corruption and public goods spending hinges on whether the revenues are above or below some threshold level. Below this threshold, the politician is constrained to spend all of the revenues and, thus, does not steal. However, she can still obtain rents by spending the revenues in exchange for bribes. In such a case in which bribery is the only source of corruption, an increase in revenues unambiguously increases public-good spending because nothing is stolen, and can decrease corruption when the marginal value of the public goods from which bribes are extracted is sufficiently high.

The threshold level of revenues thus captures, in effect, the threshold demand for public-good spending that the politician is constrained to satisfy. If government revenues are larger than the threshold level, the politician can then steal the 'extra' revenues. I find that all additional revenues above the threshold are stolen and, thus, public spending does not increase any further. Because spending does not increase, bribes are also constant. However, corruption increases in the form of theft as revenues increase above the threshold.

The implication is that the political resource curse, whereby revenues increase corruption at the expense of public-good provision, occurs because revenues from oil, natural resources, and other kinds of windfall provide revenues that exceed the threshold level that a corrupt politician would credibly spend on public goods. There exists a point at which the politician prefers to obtain rents directly by stealing revenues, rather than obtain them indirectly by spending those revenues and receiving bribes in exchange for them.

That the politician can keep public-good spending unchanged even as revenues increase implies that she can do so while remaining in office. I demonstrate that such a rent-seeking equilibrium is sustained when the politician can use the rents for political advantage. As a specific example, I consider the case when candidates in elections use both bribe-rents and/or stolen revenues to buy votes and influence electoral outcomes.

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Appendices

A Ambiguous effect of revenues on social welfare

The following proves that the total social welfare losses may increase or decrease with revenues.

Let \bar{T} denote the required threshold amount of spending, such that if actual revenues were below this, then no theft is possible. If actual revenues, T , were above the threshold, the agent obtains rents from bribes and stolen revenues, with the amount of bribes limited to the level associated with threshold spending \bar{T} , and the amount of stolen revenues equal to $T - \bar{T}$ (i.e. all revenues above the threshold are stolen).

Now consider two levels of actual revenues, T_1 and T_2 , with $T_1 < \bar{T} < T_2$. If the social welfare loss at T_2 - denote as W_2 , is greater (less) than that at T_1 - denote as W_1 , then social welfare loss increases (decreases) with actual revenues. However, I now show that $W_2 \geq W_1$, in which case the social welfare loss may increase or decrease with revenues.

Since $T_2 > \bar{T}$, the total social welfare loss at T_2 is $W_2 = (T_2 - \bar{T}) + b_{\bar{T}}$, where the first term is the amount of stolen revenues, while the second term is the amount of bribes associated with \bar{T} . Since $T_1 < \bar{T}$, the social welfare loss at T_1 is $W_1 = b_1$, where b_1 is the amount of bribes associated with T_1 .

Now, $W_2 \geq W_1$ if $(T_2 - \bar{T}) + b_{\bar{T}} \geq b_1$, or $(b_{\bar{T}} - b_1) \geq (T_2 - \bar{T})$. The RHS is positive, but the LHS is not always negative, which means that the social welfare loss at a relatively higher level of revenues, i.e. W_2 , is not always higher than the loss at a lower level of revenues, i.e. W_1 . More precisely, using equation (6): $(b_{\bar{T}} - b_1) = \frac{\lambda}{1-\lambda} [2(V(\frac{\bar{T}}{2}) - V(\frac{T_1}{2})) + (V(g_1^{T_1}) - V(g_1^{\bar{T}})) + (V(T - g_1^{T_1}) - V(T - g_1^{\bar{T}}))] \geq (T_2 - \bar{T})$.

B FOCs for g_1^{k*}

To get equation (28), differentiate (27) with respect to g_1^{A*} and set to zero to get FOC:

$$N_1 \left[\frac{\partial F}{\partial \Delta^{\mathcal{F}}} \frac{\partial \Delta^{\mathcal{F}}}{\partial g_1^{A*}} V(g_1^{A*}) + F(\Delta^{\mathcal{F}}) V'(g_1^{A*}) \right] - N_1 \frac{\partial F}{\partial \Delta^{\mathcal{F}}} \frac{\partial \Delta^{\mathcal{F}}}{\partial g_1^{A*}} V(g_1^{B*}) + \left[\frac{\mu f}{(1-\mu)e} \right] \left[\frac{N_1}{N} V'(g_1^{A*}) - \frac{N_2}{N} V'(T - g_1^{A*}) \right] = 0. \quad (50)$$

Letting $g_2^{k*} = T - g_1^{k*}$ in (24), one can get

$$\frac{\partial \Delta^{\mathcal{F}}}{\partial g_1^{A*}} = \mu f \left[\frac{N_1}{N} V'(g_1^{A*}) - \frac{N_2}{N} V'(T - g_1^{A*}) \right]. \quad (51)$$

One can then write (50) as

$$N_1 F(\Delta^{\mathcal{F}}) V'(g_1^{A*}) = \frac{\partial \Delta^{\mathcal{F}}}{\partial g_1^{A*}} \left[N_1 \frac{\partial F}{\partial \Delta^{\mathcal{F}}} [V(g_1^{B*}) - V(g_1^{A*})] - \frac{1}{(1-\mu)e} \right]. \quad (52)$$

Defining $x \equiv \frac{N_1 F(\Delta^{\mathcal{F}})}{N_1 \frac{\partial F}{\partial \Delta^{\mathcal{F}}} [V(g_1^{B*}) - V(g_1^{A*})] - \frac{1}{(1-\mu)e}}$, (52) becomes

$$x = \frac{\partial \Delta^{\mathcal{F}}}{\partial g_1^{A*}} \frac{1}{V'(g_1^{A*})}. \quad (53)$$

Finally, writing out $\frac{\partial \Delta^{\mathcal{F}}}{\partial g_1^{A*}}$ in (53) using (51) and re-arranging give FOC (28).

FOC (29) can be similarly obtained. Differentiating (27) with respect to g_1^{B*} and setting to zero give

$$\begin{aligned}
N_1 \frac{\partial F}{\partial \Delta^{\mathcal{T}}} \frac{\partial \Delta^{\mathcal{T}}}{\partial g_1^{B^*}} V(g_1^{A^*}) + N_1 V'(g_1^{B^*}) - N_1 \left[\frac{\partial F}{\partial \Delta^{\mathcal{T}}} \frac{\partial \Delta^{\mathcal{T}}}{\partial g_1^{B^*}} V(g_1^{B^*}) + F(\Delta^{\mathcal{T}}) V'(g_1^{B^*}) \right] \\
+ \left[\frac{\mu f}{(1-\mu)e} \right] \left[\frac{N_1}{N} V'(g_1^{B^*}) - \frac{N_2}{N} V'(T - g_1^{B^*}) \right] = 0.
\end{aligned} \tag{54}$$

With

$$\frac{\partial \Delta^{\mathcal{T}}}{\partial g_1^{B^*}} = -\mu f \left[\frac{N_1}{N} V'(g_1^{B^*}) - \frac{N_2}{N} V'(T - g_1^{B^*}) \right], \tag{55}$$

one can write (54) as

$$N_1 (1 - F(\Delta^{\mathcal{T}})) V'(g_1^{B^*}) = \frac{\partial \Delta^{\mathcal{T}}}{\partial g_1^{B^*}} \left[N_1 \frac{\partial F}{\partial \Delta^{\mathcal{T}}} [V(g_1^{B^*}) - V(g_1^{A^*})] + \frac{1}{(1-\mu)e} \right]. \tag{56}$$

Defining $y \equiv \frac{N_1(1-F(\Delta^{\mathcal{T}}))}{N_1 \frac{\partial F}{\partial \Delta^{\mathcal{T}}} [V(g_1^{B^*}) - V(g_1^{A^*})] + [\frac{1}{(1-\mu)e}]}$, (56) becomes

$$y = \frac{\partial \Delta^{\mathcal{T}}}{\partial g_1^{B^*}} \frac{1}{V'(g_1^{B^*})}. \tag{57}$$

Finally, writing out $\frac{\partial \Delta^{\mathcal{T}}}{\partial g_1^{B^*}}$ and re-arranging give FOC (29).

C Kuhn-Tucker conditions for $g_i^{k^*}$

To get (41), get the derivative of the Lagrangian with respect to g_1^A and set to zero:

$$N_1 \left[\frac{\partial F}{\partial \Delta} \frac{\partial \Delta}{\partial g_1^{A^*}} V(g_1^{A^*}) + F(\Delta) V'(g_1^{A^*}) \right] - N_1 \frac{\partial F}{\partial \Delta} \frac{\partial \Delta}{\partial g_1^{A^*}} V(g_1^{B^*}) + \frac{\mu f}{(1-\mu)e} \frac{N_1}{N} V'(g_1^{A^*}) - \lambda^{A^*} = 0. \tag{58}$$

Using the fact that

$$\frac{\partial \Delta}{\partial g_1^{A^*}} = \mu f \frac{N_1}{N} V'(g_1^{A^*}) - (1-\mu)e \tag{59}$$

and re-arranging give (41).

To get (42), get the derivative of the Lagrangian with respect to g_2^A and set to zero:

$$N_1 V(g_1^{A^*}) \frac{\partial F}{\partial \Delta} \frac{\partial \Delta}{\partial g_2^{A^*}} - N_1 V(g_1^{B^*}) \frac{\partial F}{\partial \Delta} \frac{\partial \Delta}{\partial g_2^{A^*}} + \frac{\mu f}{(1-\mu)e} \frac{N_2}{N} V'(g_2^{A^*}) - \lambda^{A^*} = 0 \tag{60}$$

which, with

$$\frac{\partial \Delta}{\partial g_2^{A^*}} = \mu f \frac{N_2}{N} V'(g_2^{A^*}) - (1-\mu)e, \tag{61}$$

gives (42).

To get (43), get the derivative of the Lagrangian with respect to g_1^B and set to zero:

$$N_1 \frac{\partial F}{\partial \Delta} \frac{\partial \Delta}{\partial g_1^{B^*}} V(g_1^{A^*}) + N_1 V'(g_1^{B^*}) - N_1 \left[\frac{\partial F}{\partial \Delta} \frac{\partial \Delta}{\partial g_1^{B^*}} V(g_1^{B^*}) + F(\Delta) V'(g_1^{B^*}) \right] + \frac{\mu f}{(1-\mu)e} \frac{N_1}{N} V'(g_1^{B^*}) - \lambda^{B^*} = 0 \quad (62)$$

which, with

$$\frac{\partial \Delta}{\partial g_1^{B^*}} = -\mu f \frac{N_1}{N} V'(g_1^{B^*}) + (1-\mu)e, \quad (63)$$

gives (43).

To get (44), get the derivative of the Lagrangian with respect to g_2^B and set to zero:

$$N_1 \frac{\partial F}{\partial \Delta} \frac{\partial \Delta}{\partial g_2^{B^*}} V(g_1^{A^*}) - N_1 \frac{\partial F}{\partial \Delta} \frac{\partial \Delta}{\partial g_2^{B^*}} V(g_1^{B^*}) + \frac{\mu f}{(1-\mu)e} \frac{N_2}{N} V'(g_2^{B^*}) - \lambda^{B^*} = 0 \quad (64)$$

which, with

$$\frac{\partial \Delta}{\partial g_2^{B^*}} = -\mu f \frac{N_2}{N} V'(g_2^{B^*}) + (1-\mu)e, \quad (65)$$

gives (44).

Lastly, (45) and (46) are standard complementary-slackness conditions.

D Proofs

Proposition 1

Proof. By the implicit function theorem, $\frac{dg_1^*}{dT} = -\frac{\partial F/\partial T}{\partial F/\partial g_1^*} = \frac{\lambda V''(T-g_1^*)}{V''(g_1^*) + \lambda V''(T-g_1^*)} > 0$. Since $g_1^* + g_2^* = T$, then $\frac{dg_1^*}{dT} + \frac{dg_2^*}{dT} = 1$, which implies that $\frac{dg_2^*}{dT} = \frac{V''(g_1^*)}{V''(g_1^*) + \lambda V''(T-g_1^*)} > 0$. Thus, (a), (b), and (c) are obtained by comparing $\frac{\lambda V''(T-g_1^*)}{V''(g_1^*) + \lambda V''(T-g_1^*)}$ with $\frac{\lambda V''(g_1^*)}{V''(g_1^*) + \lambda V''(T-g_1^*)}$ or, simplifying, λ with $\frac{V''(g_1^*)}{V'(T-g_1^*)}$. ■

Remark Note that the second-order condition (SOC) for a maximum is $V''(g_1^*) + \lambda V''(T-g_1^*) < 0$, which is met since $V''(\cdot) < 0$. The SOC does not restrict the relative magnitudes of $V''(g_1^*)$ and $V''(T-g_1^*)$, as it only implies that $\lambda > \frac{V''(g_1^*)}{-V''(T-g_1^*)}$, which always holds, i.e. in both cases (a) and (b).

One can also show that $\frac{dg_1^*}{dT} \geq \frac{dg_2^*}{dT}$ by demonstrating that $\frac{dg_1^*}{dT} \geq \frac{1}{2}$. Since $g_1^* + g_2^* = T$, one can write $g_1^* = \alpha T$ and $g_2^* = (1-\alpha)T$. With $g_1^* > g_2^*$, it must be that $\alpha \in (\frac{1}{2}, 1)$. Thus, if α is a constant, $\frac{dg_1^*}{dT} = \alpha > \frac{1}{2}$. However, more generally, $g_1^* = \alpha(\lambda, T)T$, with $\alpha(\lambda, T) > \frac{1}{2}$, in which case $\frac{dg_1^*}{dT} = \frac{\partial \alpha(\lambda, T)}{\partial T} T + \alpha(\lambda, T)$. Thus, $\frac{dg_1^*}{dT} \geq \frac{1}{2}$, since $\frac{1}{2} < \alpha(\lambda, T) \leq \frac{1}{2} - \frac{\partial \alpha(\lambda, T)}{\partial T}$ if $\frac{\partial \alpha(\lambda, T)}{\partial T} \neq 0$.

A special case of $g_1^* = \alpha(\lambda, T)T$ is $g_1^* = f(\lambda)T^n$, where $f(\lambda) > 0$, and either $n \in (0, 1)$ or $n > 1$. In this case, $\frac{dg_1^*}{dT} = f(\lambda)(nT^{n-1}) \geq \frac{1}{2}$.²⁴ For an example in which $n \in (0, 1)$, suppose $g_1^* = \frac{\sqrt{T}}{1+\lambda}$, which

²⁴Note that n is non-negative since if $n < 0$, then $g_1^* = f(\lambda)\frac{1}{T^n}$, which implies $\frac{dg_1^*}{dT} = f(\lambda)\left(\frac{-nT^{n-1}}{T^{2n}}\right) < 0$. This is not possible, as (the proof of) Proposition 1 has shown that $\frac{dg_1^*}{dT} > 0$.

implies $g_2^* = \frac{(1+\lambda)T - \sqrt{T}}{1+\lambda}$. Then $\frac{dg_1^*}{dT} = \frac{1}{2(1+\lambda)\sqrt{T}} \geq \frac{1}{2}$, since $\frac{1}{\sqrt{T}} \geq 1 + \lambda$. An example in which $n > 1$ is $g_1^* = \frac{T^2}{1+\lambda}$, which implies $g_2^* = \frac{(1+\lambda)T - T^2}{1+\lambda}$, and where $T \in (0, (1 + \lambda))$. In this case, $\frac{dg_1^*}{dT} = \frac{2T}{1+\lambda} \geq \frac{1}{2}$, since $T \geq \frac{1+\lambda}{4}$. ■

Proposition 2

Proof. Differentiating (6) with respect to T gives $\frac{\partial b^*}{\partial T} = \frac{\lambda}{1-\lambda} \left[V'(\frac{T}{2}) - V'(g_1^*) \frac{dg_1^*}{dT} - V'(T - g_1^*) (1 - \frac{dg_1^*}{dT}) \right]$, which is greater than zero if $V'(\frac{T}{2}) > V'(g_1^*) \frac{dg_1^*}{dT} + V'(T - g_1^*) (1 - \frac{dg_1^*}{dT})$ or, rearranging, $\frac{dg_1^*}{dT} < \frac{V'(\frac{T}{2}) - V'(T - g_1^*)}{V'(g_1^*) - V'(T - g_1^*)}$. Items (b) and (c) directly follow. ■

Proposition 3

Proof. I apply the implicit function theorem to the system of equations (9), (10), (11). Focusing on interior solutions, necessary for g_1^*, g_2^*, γ^* to exist is that the inverse of

$$A = \begin{pmatrix} V''(g_1^*) & \frac{\lambda}{1-\lambda} V''(g_2^*) & -1 \\ 0 & \frac{\lambda}{1-\lambda} V''(g_2^*) & -1 \\ \gamma^* & \gamma^* & g_1^* + g_2^* - T \end{pmatrix} \quad (66)$$

exists or, equivalently, that the determinant of A is non-zero. Note that $\det A = V''(g_1^*) [\frac{\lambda}{1-\lambda} V''(g_2^*) (g_1^* + g_2^* - T) + \gamma^*]$. If theft occurs in equilibrium, then $g_1^* + g_2^* - T < 0$, which implies (from equation (11)) that $\gamma^* = 0$. Imposing $\gamma^* = 0$, $\det A = V''(g_1^*) [\frac{\lambda}{1-\lambda} V''(g_2^*) (g_1^* + g_2^* - T)]$, which is less than zero, unless $g_1^* = 0$ or $g_2^* = 0$ in which case $\det A = 0$. Thus, if sufficiency conditions are met such that g_1^*, g_2^*, γ^* exist, it must be that when theft occurs such that $\gamma^* = 0$, some revenues are allocated to both principals, i.e. $g_1^*, g_2^* > 0$. ■

Proposition 4

Proof. Applying Cramer's rule,

$$\frac{dg_1^*}{dT} = -\frac{1}{\det A} \det \begin{pmatrix} V'(g_1^*) \frac{dg_1^*}{dT} \frac{1}{1-\lambda} - \frac{d\gamma^*}{dT} & 0 & -1 \\ V'(g_2^*) \frac{dg_2^*}{dT} \frac{1}{1-\lambda} - \frac{d\gamma^*}{dT} & \frac{\lambda}{1-\lambda} V''(g_2^*) & -1 \\ (g_1^* + g_2^* - T) \frac{d\gamma^*}{dT} - \gamma^* & \gamma^* & g_1^* + g_2^* - T \end{pmatrix}. \text{ Now if theft occurs in equi-}$$

librium, $g_1^* + g_2^* < T$, which by equation (11) implies $\gamma^* = 0$ and, hence, $\frac{d\gamma^*}{dT}$. Imposing $\gamma^* = 0$ and $\frac{d\gamma^*}{dT}$ gives $\frac{dg_1^*}{dT} = -\frac{1}{\det A} [(g_1^* + g_2^* - T) V'(g_1^*) \frac{dg_1^*}{dT} \frac{1}{1-\lambda} V''(g_2^*)]$ or, simplifying, $\frac{dg_1^*}{dT} = 0$. Analogously,

$$\frac{dg_2^*}{dT} = -\frac{1}{\det A} \det \begin{pmatrix} V''(g_1^*) \frac{1}{1-\lambda} & V'(g_1^*) \frac{dg_1^*}{dT} \frac{1}{1-\lambda} - \frac{d\gamma^*}{dT} & -1 \\ 0 & V'(g_2^*) \frac{dg_2^*}{dT} \frac{1}{1-\lambda} - \frac{d\gamma^*}{dT} & -1 \\ \gamma^* & (g_1^* + g_2^* - T) \frac{d\gamma^*}{dT} - \gamma^* & g_1^* + g_2^* - T \end{pmatrix}.$$

Imposing $\gamma^* = 0$ and $\frac{d\gamma^*}{dT} = 0$ gives $\frac{dg_2^*}{dT} = -\frac{1}{\det A} [(g_1^* + g_2^* - T) V'(g_2^*) \frac{dg_2^*}{dT} \frac{1}{1-\lambda} V''(g_1^*)]$ or, simplifying, $\frac{dg_2^*}{dT} = 0$. ■

Lemma 1

I first show that $g_1^0, g_2^0 > 0$ by applying the implicit function theorem to the system of equations (13), (14) and (15). That is, it is necessary that $\det B = \begin{pmatrix} \lambda V''(g_1^0) & 0 & -1 \\ 0 & \lambda V''(g_2^0) & -1 \\ \gamma^0 & \gamma^0 & g_1^0 + g_2^0 - T \end{pmatrix}$ is non-zero. Imposing $\gamma^0, \frac{d\gamma^0}{dT} = 0$ and evaluating, $\det B = (g_1^0 + g_2^0 - T)\lambda V''(g_1^0)\lambda V''(g_2^0)$ which is less than zero, unless $g_1^0 = 0$ or $g_2^0 = 0$. That is, assuming sufficiency conditions are met such that g_1^0, g_2^0, γ^0 exist, some spending is still allocated, i.e. $g_1^0, g_2^0 > 0$ even when theft occurs (i.e. $\gamma^0 = 0$).

To prove Lemma 1:

Proof. The proof is similar to the proof of Proposition 2.4. Applying Cramer's rule,

$$\frac{dg_1^0}{dT} = -\frac{1}{\det B} \det \begin{pmatrix} \lambda V''(g_1^0) \frac{dg_1^0}{dT} & 0 & -1 \\ \lambda V''(g_2^0) \frac{dg_2^0}{dT} & \lambda V''(g_2^*) & -1 \\ 0 & 0 & g_1^0 + g_2^0 - T \end{pmatrix}. \text{ Now if theft occurs in equilibrium,}$$

$g_1^0 + g_2^0 < T$, which by equation (16) implies $\gamma^0 = 0$ and, hence, $\frac{d\gamma^0}{dT} = 0$. Imposing $\gamma^0 = 0$ and $\frac{d\gamma^0}{dT} = 0$ gives $\frac{dg_1^0}{dT} = -\frac{1}{\det B} [(g_1^0 + g_2^0 - T)\lambda V''(g_1^0)\frac{dg_1^0}{dT}\lambda V''(g_2^0)]$ or, simplifying, $\frac{dg_1^0}{dT} = 0$. Analogously,

$$\frac{dg_2^0}{dT} = -\frac{1}{\det B} \det \begin{pmatrix} \lambda V''(g_1^0) & \lambda V''(g_1^0) \frac{dg_1^0}{dT} & -1 \\ 0 & \lambda V''(g_2^0) \frac{dg_2^0}{dT} & -1 \\ 0 & 0 & g_1^0 + g_2^0 - T \end{pmatrix}. \text{ Imposing } \gamma^0 = 0 \text{ and } \frac{d\gamma^0}{dT} = 0 \text{ gives}$$

$$\frac{dg_2^0}{dT} = -\frac{1}{\det B} [(g_1^0 + g_2^0 - T)\lambda V''(g_1^0)\lambda V''(g_2^0)\frac{dg_2^0}{dT}] \text{ or, simplifying, } \frac{dg_2^0}{dT} = 0. \quad \blacksquare$$

Proposition 5

Proof. Differentiating (16) with respect to T gives $\frac{\partial b^*}{\partial T} = \frac{\lambda}{1-\lambda} [V'(g_1^0)\frac{dg_1^0}{dT} + V'(g_2^0)\frac{dg_2^0}{dT} - V'(g_1^*)\frac{dg_1^*}{dT} - V'(g_2^*)\frac{dg_2^*}{dT}] - \frac{dg_1^0}{dT} - \frac{dg_2^0}{dT} + \frac{dg_1^*}{dT} + \frac{dg_2^*}{dT}$. By Lemma 1, $\frac{dg_1^0}{dT}, \frac{dg_2^0}{dT} = 0$, and by Proposition 4, $\frac{dg_1^*}{dT}, \frac{dg_2^*}{dT} = 0$. Thus, $\frac{\partial b^*}{\partial T} = 0$. ■

Corollary 1

Proof. If theft occurs in equilibrium, then $R^* = T - g_1^* - g_2^* + b^*$. Differentiating with respect to T gives $\frac{\partial R^*}{\partial T} = 1 - \frac{dg_1^*}{dT} - \frac{dg_2^*}{dT} + \frac{\partial b^*}{\partial T}$, which is equal to 1 by Propositions 4 and 5. ■

Proposition 6

Proof. From corollary 1, $\frac{\partial R^*}{\partial T} = 1$. Note that $R^{\mathcal{F}}$ is simply the amount of bribes when no theft is possible, and is thus given by equation (6). Thus, $\frac{\partial R^{\mathcal{F}}}{\partial T}$ is equal to the expression for $\frac{\partial b^*}{\partial T}$ given in the proof of Proposition 2. Comparing such expression with 1 leads to items (a), (b), and (c). ■

Proposition 7

Proof. Note that if $\alpha^{k^*} > 0$, then $g_1^{k^*} < T$. For $\alpha^{A^*} > 0$, it must be that $\mu f \frac{N_1}{N} - x > 0$, and for $\alpha^{B^*} > 0$, it must be that $\mu f \frac{N_1}{N} + y > 0$. One can write (28) as (i) $\mu f \frac{N_2}{N} = (\mu f \frac{N_1}{N} - x) \frac{V'(g_1^{A^*})}{V'(T-g_1^{A^*})}$, and (29) as (ii) $\mu f \frac{N_2}{N} = (\mu f \frac{N_1}{N} + y) \frac{V'(g_1^{B^*})}{V'(T-g_1^{B^*})}$. Equating (i) and (ii) and re-arranging give $\frac{\mu f \frac{N_1}{N} - x}{\mu f \frac{N_1}{N} + y} = \frac{V'(T-g_1^{A^*})}{V'(g_1^{A^*})} \frac{V'(g_1^{B^*})}{V'(T-g_1^{B^*})}$. Since the RHS is non-negative, and $\mu f \frac{N_1}{N} \neq x$, $\mu f \frac{N_1}{N} \neq -y$, then $\mu f \frac{N_1}{N} - x > 0$ and $\mu f \frac{N_1}{N} + y > 0$. ■

Proposition 8

Proof. Note that $g_1^{A^*} \geq g_1^{B^*}$ if $\alpha^{A^*} \leq \alpha^{B^*}$, and vice-versa. In turn, $\alpha^{A^*} = \frac{\mu f \frac{N_2}{N}}{\mu f \frac{N_1}{N} - x} \leq \frac{\mu f \frac{N_2}{N}}{\mu f \frac{N_1}{N} + y} = \alpha^{B^*}$ if $y \leq -x$. Writing out the expressions for y and x , this condition becomes $\frac{N_1 \frac{\partial F}{\partial \Delta^T} [V(g_1^{B^*}) - V(g_1^{A^*})] - [\frac{1}{(1-\mu)e}]}{N_1 \frac{\partial F}{\partial \Delta^T} [V(g_1^{B^*}) - V(g_1^{A^*})] + [\frac{1}{(1-\mu)e}]} \leq \frac{-F(\Delta^T)}{1-F(\Delta^T)}$. To simplify, let $a \equiv N_1 \frac{\partial F}{\partial \Delta^T} [V(g_1^{B^*}) - V(g_1^{A^*})]$ and $b \equiv \frac{1}{(1-\mu)e}$. Then the condition can be written as $\frac{a-b}{a+b} \leq \frac{-F(\Delta^T)}{1-F(\Delta^T)}$, which simplifies to $F(\Delta^T) \leq \frac{1}{2} - \frac{a}{2b}$, the RHS of which has been defined as z . (Result (iii) corresponds to $F(\Delta^T) = \frac{1}{2} - \frac{a}{2b}$). ■

Lemma 2

Proof. Writing out the expression for α^{A^*} and comparing with $\frac{N_2}{N_1}$ give: $\alpha^{A^*} = \frac{\mu f \frac{N_2}{N}}{\mu f \frac{N_1}{N} - x} \geq \frac{N_2}{N_1}$ or, simplifying, $x \geq 0$. (It follows that $x = 0 \iff \alpha^{A^*} = \frac{N_2}{N_1}$.) Similarly, $\alpha^{B^*} = \frac{\mu f \frac{N_2}{N}}{\mu f \frac{N_1}{N} + y} \geq \frac{N_2}{N_1}$ or, simplifying, $y \leq 0$. (It follows that if $y = 0 \iff \alpha^{B^*} = \frac{N_2}{N_1}$.) ■

Lemma 3

Proof. Note first that the numerators from the expressions for x and y are non-zero and positive since $F(\Delta^T) \in (0, 1)$. Thus, whether $x, y \geq 0$ depend on their respective denominators. For $x \geq 0$, it must be that $V(g_1^{B^*}) - V(g_1^{A^*}) \geq \frac{1}{(1-\mu)eN_1} \frac{\partial F}{\partial \Delta^T} \equiv w$, where the RHS is greater than zero. Thus, when $g_1^{B^*} \leq g_1^{A^*}$, the LHS is less than or equal to zero, which implies $x < 0$. If $g_1^{B^*} > g_1^{A^*}$, then the LHS is greater than zero. In this case, one compares $V(g_1^{B^*}) - V(g_1^{A^*})$ with w . An analogous argument can be made to establish whether $y \geq 0$, which now requires $V(g_1^{A^*}) - V(g_1^{B^*}) \leq w$. (Note that in this case, the LHS is less than or equal to zero when $g_1^{B^*} \geq g_1^{A^*}$, which implies $y > 0$.) ■

Proposition 9

Proof. To prove (i), note that Proposition 8 establishes that $g_1^{A^*} = g_1^{B^*} \iff F(\Delta^T) = z$. From Lemma 3, $x < 0$ and $y > 0 \iff g_1^{A^*} = g_1^{B^*}$. Finally, from Lemma 2, $\alpha^{A^*} < \frac{N_2}{N_1} \iff x < 0$ and

$\alpha^{B^*} < \frac{N_2}{N_1} \iff y > 0$. Thus, both A and B attach higher weight to group 1's, than to group 2's utility, relative to $\frac{N_2}{N_1}$, which is the weight implied by the social optimum. Hence, $g_1^{A^*} = g_1^{B^*} > g_1^0$. Results (ii) and (iii) are analogously obtained using Proposition 8, and Lemmas 2 and 3. ■

Proposition 10

Proof. One can subtract $V(g_1^0)$ from both sides of the FOC for $g_1^{k^*}$ to get (i) $V(g_1^{k^*}) - V(g_1^0) = \alpha^{k^*} V(T - g_1^{k^*}) - V(g_1^0)$. Then, using the FOC for g_1^0 , one can plug into the RHS of (ii) an expression for $V(g_1^0)$: (ii) $V(g_1^{k^*}) - V(g_1^0) = \alpha^{k^*} V(T - g_1^{k^*}) - \frac{N_2}{N_1} V(T - g_1^0)$. Finally, substituting the RHS of (ii) into the LHS of the condition for $b^{k^*} \geq 0$ gives $\alpha^{k^*} V(T - g_1^{k^*}) - \frac{N_2}{N_1} V(T - g_1^0) \leq \frac{N_2}{N_1} [V(T - g_1^0) - V(T - g_1^{k^*})]$, which reduces to $\frac{V(T - g_1^0)}{V(T - g_1^{k^*})} \geq \frac{\alpha^{k^*} N_1 + N_2}{2N_2}$. ■

Proposition 11

Proof. From equation (33), $b^{A^*} \geq b^{B^*}$ if $V(g_1^{B^*}) - V(g_1^{A^*}) \geq \frac{(1-\mu)e}{\mu f} \frac{N_2}{N_1} [V(T - g_1^{A^*}) - V(T - g_1^{B^*})]$. The LHS of the inequality is ≤ 0 , while the RHS is ≥ 0 , when $g_1^{A^*} \geq g_1^{B^*}$. Thus, $b^{A^*} \leq b^{B^*}$ if $g_1^{A^*} \geq g_1^{B^*}$. (Both the LHS and RHS are equal to zero when $g_1^{A^*} = g_1^{B^*}$, which implies $b^{A^*} = b^{B^*}$.) The reverse holds – if $g_1^{A^*} \geq g_1^{B^*}$, then $V(g_1^{B^*}) - V(g_1^{A^*}) \leq \frac{(1-\mu)e}{\mu f} \frac{N_2}{N_1} [V(T - g_1^{A^*}) - V(T - g_1^{B^*})]$, which means $b^{A^*} \leq b^{B^*}$. ■

Proposition 12

Proof. By assumption, $V'(g_i^{k^*}) \geq 0$, while equations (i) and (ii) underlying (47) require that $V'(g_i^{k^*}) \neq 0$. Thus, it must be that $V'(g_i^{k^*}) > 0$, which implies that $g_i^{k^*} > 0$. ■

Lemma 4

Proof. By proposition 12, the LHS of (47) is greater than zero. For the RHS to be greater than zero, $\frac{\partial g_2^{k^*}}{\partial g_1^{k^*}} \neq |1|$ and $\frac{\partial g_2^{A^*}}{\partial g_1^{A^*}} \geq 1 \iff \frac{\partial g_2^{B^*}}{\partial g_1^{B^*}} \leq 1$. ■

Proposition 13

Proof. Note that $g_1^{A^*} \geq g_1^{B^*} \iff V'(g_1^{A^*}) \geq V'(g_1^{B^*})$, and $g_1^{A^*} = g_1^{B^*} \iff V'(g_1^{A^*}) = V'(g_1^{B^*})$. By (47), $V'(g_1^{A^*}) \geq V'(g_1^{B^*}) \iff (1 - F(\Delta)) \left(\frac{\partial g_2^{A^*}}{\partial g_1^{A^*}} - 1 \right) \geq F(\Delta) \left(1 - \frac{\partial g_2^{B^*}}{\partial g_1^{B^*}} \right)$ or, re-arranging: $F(\Delta) \leq \left(\frac{\partial g_2^{A^*}}{\partial g_1^{A^*}} - 1 \right) \left(\frac{\partial g_2^{A^*}}{\partial g_1^{A^*}} - \frac{\partial g_2^{B^*}}{\partial g_1^{B^*}} \right) \equiv w$, while $V'(g_1^{A^*}) = V'(g_1^{B^*}) \iff F(\Delta) = w$. ■

Proposition 14

Proof. The socially optimal allocation, i.e. when there is no theft or bribery such that $\overline{U}^k = \mu f[\frac{N_1}{N}V(g_1^k) + \frac{N_2}{N}V(T - g_1^k)]$ is maximized, satisfies $\frac{V'(g_1^0)}{V'(T-g_1^0)} = \frac{N_2}{N_1}$. Now the allocation of each candidate is such that $\frac{V'(g_1^k)}{V'(g_2^k)} \geq \frac{V'(g_1^0)}{V'(T-g_1^0)}$. To see this, recall that $\frac{\partial g_2^{A*}}{\partial g_1^{A*}} = \frac{\partial \Delta}{\partial g_1^{A*}} \frac{\partial g_2^{A*}}{\partial \Delta} = \frac{\mu f \frac{N_1}{N} V'(g_1^{A*}) - (1-\mu)e}{\mu f \frac{N_2}{N} V'(g_2^{A*}) - (1-\mu)e}$. By Lemma 4, $\mu f \frac{N_1}{N} V'(g_1^{A*}) - (1-\mu)e \geq \mu f \frac{N_2}{N} V'(g_2^{A*}) - (1-\mu)e$ or, simplifying, $\frac{V'(g_1^{A*})}{V'(g_2^{A*})} \geq \frac{N_2}{N_1}$. Similarly, $\frac{\partial g_2^{B*}}{\partial g_1^{B*}} = \frac{\partial \Delta}{\partial g_1^{B*}} \frac{\partial g_2^{B*}}{\partial \Delta} = \frac{-\mu f \frac{N_1}{N} V'(g_1^{B*}) + (1-\mu)e}{-\mu f \frac{N_2}{N} V'(g_2^{B*}) + (1-\mu)e}$ and, by Lemma 4, $-\mu f \frac{N_1}{N} V'(g_1^{B*}) + (1-\mu)e \leq -\mu f \frac{N_2}{N} V'(g_2^{B*}) + (1-\mu)e$ or $\frac{V'(g_1^{B*})}{V'(T-g_1^{B*})} \geq \frac{N_2}{N_1}$. Substituting in for $\frac{N_2}{N_1}$, we have that $\frac{V'(g_1^k)}{V'(g_2^k)} \geq \frac{V'(g_1^0)}{V'(T-g_1^0)}$ for each $k = \{A, B\}$. ■

Proposition 15

Proof. For $\frac{V'(g_1^{k^0})}{V'(g_2^{k^0})} = \frac{N_2}{N_1}$ to hold for each candidate A and B, it must be that $g_1^{A^0} = g_1^{B^0} \iff g_2^{A^0} = g_2^{B^0}$. In this case, (49) reduces to $\left[\frac{1}{(1-\mu)e}\right] \left[\mu f \left[\frac{N_1}{N}(V(g_1^{B^*}) - V(g_1^{A^*})) + \frac{N_2}{N}(V(g_2^{B^*}) - V(g_2^{A^*}))\right]\right] + g_1^{A^*} + g_2^{A^*} - (g_1^{B^*} + g_2^{B^*})$. Thus, $b_T^{A^*} \geq b_T^{B^*}$ if $\left[\frac{1}{(1-\mu)e}\right] \left[\mu f \left[\frac{N_1}{N}(V(g_1^{B^*}) - V(g_1^{A^*})) + \frac{N_2}{N}(V(g_2^{B^*}) - V(g_2^{A^*}))\right]\right] + g_1^{A^*} + g_2^{A^*} - (g_1^{B^*} + g_2^{B^*}) \geq 0$ or, re-arranging: $[(g_1^{B^*} + g_2^{B^*}) - (g_1^{A^*} + g_2^{A^*})] \leq \left[\frac{\mu f}{(1-\mu)e}\right] \left[\frac{N_1}{N} V(g_1^{B^*}) + \frac{N_2}{N} V(g_2^{B^*})\right] - \left[\frac{N_1}{N} v(g_1^{A^*}) + \frac{N_2}{N} V(g_2^{A^*})\right]$, or $x \leq \left[\frac{\mu f}{(1-\mu)e}\right] y$. (Result (iii) immediately follows.) ■